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RELIABILITY-GROWTH ASSESSMENT, PREDICTION, AND CONTROL FOR ELECTRONIC ENGINE CONTROL (GAPCEEC)

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This Reliability-Growth Assessment, Prediction and Control for Electronic Engine Control (GAPCEEC) program was performed under contract F33615-81-C-2015. This 22-month program was formulated to study and analyze the reliability-growth phenomena obtained from electronic and hydromechanical gas turbine control equipment. This study consisted of a thorough review of the literature; an evaluation of five math models against four gas turbine control components having field and development experience; selection of the optimum reliability growth model; and development of a procedure for applying the optimum model to future development programs.

The conclusions of this program were: (1) the AMSAA/Duane model is the best model; however, the Endless-Burn-In model and Time-Series Analysis were also considered acceptable; (2) that AMSAA model parameters should be estimated via the method of maximum-likelihood; and (3) that data should be tracked continuously on an individual and fleet basis.

PREFACE

This final report was prepared by United Technologies Corporation, Pratt & Whitney, Engineering Division, Florida Operations. As part of the contractual requirement, this document will report the work conducted from 1 December 1981 to 1 November 1983 on Air Force Contract F33615-81-C-2015, Reliability-Growth Assessment, Prediction and Control for Electronic Engine Control (GAPCEEC).

The Program is being sponsored by AFWAL/POTC/USAF, Wright-Patterson AFB, Ohio 45433. Mr. Paul Adams is the Air Force Project Engineer and M.E. McGlone is the P&W Program Manager.

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LIST OF SYMBOLS

a_t and ε_t	Independent identically distributed random variable with mean 0 and variance at time t; referred to as the *white noise.*
AMF	~ Air Maintenance Form
AMSAA	~ Army Material Systems Analysis Agency
ANOVA	~ Analysis of variance
ARIMA	~ Auto-Regressive-Integrated-Moving Average
B^k	Backward shift operator; e.g., $B^k \cdot \lambda_c(t) = \lambda_c(t-k)$
C_k	Estimate of γ_k (covariance between two cumulative failure rates)
CCRA	~ Critical Component Reliability Assessment
CERT	~ Combined Environment Reliability Test
CIVV	~ Compressor Inlet Variable Vane
DEEC	~ Digital Electronic Engine Control
e	~ constant = 2.7182818284 . . .
EBI	~ Endless-Burn-In
EEC	~ Electronic Engine Control
EHR	~ Event History Recorder
F(x)	~ cumulative distribution function evaluated at x ($P(y < x) = F(x)$)
j failures ε (o,t)	~ j failures in the interval from (o,t).
K!	~ $K \cdot (K-1) \cdot (K-2) \ldots \cdot (2)$
K,A,B,	~ parameters associated with a Deterministic Model
LS	~ Least Square
m(t)	~ mean value function of a nonhomogeneous Poisson process at time t
MFPS	~ Main Fuel Pump System
MTBF	~ Mean Time Between Failures
N(t)	~ Cumulative incidence by time t
NHPP	~ Nonhomogeneous Poisson Process

$p(t)$	~ intensity function of a nonhomogeneous Poisson process at time t	
$P(x = k)$	~ probability that x is equal to k	
RMSE	~ Root Mean Square Error	
SA-ALC	~ San Antonio Air Logistics Center	
T_{ave}	~ Average fleet time (EBI model)	
TT2.5	~ Total Temperature at Fan Exit	
$W_c^d(t)$	d^{th} difference of the cumulative failure rate function ($\lambda_c(t)$):	
	e.g., For d = 1, $W_c^1(t) = \lambda_c(t) - \lambda_c(t-1)$	
$X^2_{(K_1,\alpha)}$	~ Chi-Square distribution with K degrees of freedom evaluated from o, t α (tabled values)	
λ	~	Parameters of Growth Models
β	~	
θ	~	
α	~	
γ	~	
$\sum_i^j$	~ Sum all terms from i to j:	
	e.g., $\sum_{i=1}^{3} x_i = x_1 + x_2 + x_3$	
π_i^j	~ multiply all terms from i to j:	
	e.g., $\pi_{i=1}^{3} x_i = x_1 \cdot x_2 \cdot x_3$	
$\hat{\lambda}$	~ maximum likelihood Estimates of the Model Parameter λ	
$\hat{\beta}$	~ maximum likelihood Estimates of the Model Parameter β	
$\hat{\gamma}$	~ maximum likelihood Estimates of the Model Parameter γ	
$\hat{\alpha}$	~ maximum likelihood Estimates of the Model Parameter α	
$\hat{\theta}$	~ maximum likelihood Estimates of the Model Parameter θ	
$\frac{\delta M(t)}{\delta \lambda}$	~ the derivative of the function M(t) with respect to the parameter λ	
λ_i	~ Instantaneous failure rate (EBI model)	

λ_r	~	Limiting failure rate for λ_i (EBI model)
$\bar{\beta}$	~	An unbiased estimate of the model parameter β
$\lambda_c(t)$ and $\lambda_{c,t}$		Cumulative failure rate function at time t.
μ		Mean of the cumulative failure rate
ψ_i		Moving average coefficients which express the cumulative failure rate as an infinite weighted sum of white noise random variables.
$\varphi_1, \varphi_2, \ldots \varphi_p$		p autoregressive parameters in the Time Series model: $w_c^d(t) = \varphi_1 w_c^d(t-1) + \varphi_2 w_c^d(t-2) + \ldots + \varphi_p w_c^d(t-p) + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} \ldots - \theta_q a_{t-q}$
$\theta_1, \theta_2, \theta_q$		q moving average parameters in the Time Series model: $w_c^d(t) = \varphi_1 w_c^d(t-1) + \varphi_2 w_c^d(t-2) + \ldots + \varphi_p w_c^d(t-p) + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} \ldots - \theta_q a_{t-q}$
$\varphi(B)$		Backward shift function of the p autoregressive parameters $\varphi(B) = 1 - \varphi_1 B^1 - \varphi_2 B^2 \ldots - \varphi_p B^P$
$\theta(B)$		Backward shift function of the q moving average parameters: $\theta(B) = 1 - \theta_1 B^1 - \theta_2 B^2 \ldots - \theta_q B^q$
θ_0		Drift parameter of the general Time Series model: $\varphi(B)W_c^d(t) = \theta_0 + \theta(B)a_t$
π_i		Autoregressive coefficients which express the cumulative failure rate as an infinite weighted sum of previous cumulative failure rates
γ_k		Covariance between two cumulative failure rates at lag k
ρ_k		Autocorrelation between two cumulative failure rates at lag k
r_k		Estimate of ρ_k (autocorrelation between two cumulative failure rates)
$\bar{\lambda}_c(t)$		Mean of the cumulative failure rates
σ_a^2		Variance of the white noise random variables
$\sigma(w_c^1(t))$		Standard deviation of the difference of cumulative failure rates

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SUMMARY

This report details the analysis and study procedures used during the GAPCEEC program.

- *Section I — Introduction* — Summarizes the program objectives, background, and reasons for conducting the program; and, provides an overview of program structure, analysis results, and conclusions.

- *Section II — Reliability-Growth Model Research and Selection* — This section discusses the literature search, dialogue with the specialist, initial criteria for comparing models, and models selected for evaluation.

- *Section III — Evaluation of Reliability-Growth Models and Selection of the Best Model* — This section discusses the components that were selected for evaluation, the data used, the methodology used to evaluate the models, and the results of the evaluation. The AMSAA/Duane model was selected as the best model.

- *Section IV — Program Synthesis* — This section discusses the confidence limits for the AMSAA/Duane model and the Time Series model. It also gives an overview of the procedure (contained in Appendix D to the report) for applying the AMSAA/Duane model.

- *Section V — Warranty Considerations* — This section discusses the application of the AMSAA/Duane reliability-growth model to warranty evaluation.

- *Section VI — Results and Conclusions* — This section summarizes the significant results of this study.

- *Appendix A — List of Applicable Documents* — Presents the results of the literature search of books, articles, and reports related to reliability-growth modeling.

- *Appendix B — Criteria for Reliability-Growth Model Selection* — Criteria used for evaluating the selected growth models for determining the optimum model.

- *Appendix C — Maximum Likelihood Estimation of Parameters of Reliability-Growth Models* — Illustrates the mathematical derivation of the maximum likelihood estimators for the AMSAA model, Cox-Lewis model, and Modified Duane model.

- *Appendix D — Growth Model Application Procedure* — Contains the procedure for applying the AMSAA/Duane model to electronic control development programs.

- *Appendix E — Time Series Analysis* — Contains the procedure for applying the Box-Jenkins Time Series Analysis to reliability-growth situations.

SECTION I

INTRODUCTION

A. BACKGROUND

The use of digital electronic controls for gas turbine engines has become an established goal throughout the propulsion industry. Most gas turbine engine manufacturers, in conjunction with Government agencies, are evaluating various concepts for full authority electronic control of their engines. These electronic systems will be required to surpass the reliability demonstrated by existing hydromechanical systems, which becomes a new role for modern electronics in that safety-of-flight and engine operational readiness will be impacted directly. That is, engines so equipped will be safe and effective to operate only so long as the reliability and integrity of the electronics remain intact. Ultimately, the entire weapons system will have all of its control systems working in an integrated fashion and functioning synergistically rather than individually. High mission reliability will then be accomplished through redundancy with a high confidence factor of fail-operational capability. High individual component reliabilities will be absolutely necessary to accomplish this goal within acceptable cost and maintainability limits.

There is probably no single best approach to high reliability, since each application is likely to have unique requirements. However, reliability-growth modeling is a common element of various approaches which can be useful for diverse kinds of equipment and systems. In fact, a significant amount of information accumulated in technical literature over the past 2 decades strongly indicates that the reliability-growth modeling concept can be a very effective tool for the assessment, prediction and control of reliability in modern electronic equipments and systems.

B. PROBLEM

The ability to predict the reliability of a control system/component population within reasonably close limits, at any point in its development and operational service life, is a significant requisite to achieving the very high reliability goals set forth for future engine control systems. At any point in time, both the achieved reliability and its growth trend must be identifiable quantitatively to permit timely corrective action to be taken when it appears likely that specified reliability goals will not be met on schedule.

C. OBJECTIVE

Two objectives of this program were to: (1) select the optimum reliability growth model for application to future electronic engine control system components and (2) to evolve a reliability-growth modeling analytical procedure which would provide guidance with which to confidently assess, predict, and control the reliability growth of military engine control systems and components throughout their development and service lives. In addition, an assessment was to be made as to the applicability of the program results in establishing the feasibility and validity of applying warranty provisions to future development programs for military aircraft engines.

D. APPROACH

A technical team headed by Pratt & Whitney (P&W), including team members from Hamilton Standard and Chandler Evans Corporations, was assembled to pursue this objective, along with frequent coordination with leaders in the reliability-growth modeling field. This team included personnel with extensive backgrounds in control system design and integration, statistical analysis and reliability modeling.

Guidance required to analyze and predict the reliability growth which can be expected was obtained by reviewing available data sources and reliability analysis tools. Experience with the F100 EEC, which has accumulated over 1 million operating hours in the United States Air Force (USAF) inventory, provided insight into many areas where better tools for reliability analysis and predictions are required; including insight into the assets and liabilities of current reliability data tracking systems. These systems have made available a substantial data base of USAF, contractor, and vendor reliability data.

Many reliability models and statistical analysis tools are available which may be used to address the problems of predicting and assessing the reliability growth of gas turbine control systems and components with varying degrees of success.

This program was structured to review the literature on these various methods, provide a dialogue with experts from the field, and evaluate and select a model which will best meet the needs of the control components of the future. In the process of evaluating modeling techniques, due regard was placed on the sources and validity of the data sets, accuracy of the results, and feasibility of the models to evaluate future fault tolerant redundant control systems.

E. SYNOPSIS

During this program, more than 20 models of 4 major classes, along with more than 200 books, articles, and reports that deal with reliability-growth modeling, were selected for review. Dialogue was established with several specialists in growth modeling.

Criteria were established, involving applicability, flexibility, ease of use, and short-and long-term forecasting precision, for model comparisons. Five models were selected for further testing with data bases acquired from development testing and field service of components deployed in operational service. These models were:

- AMSAA (Army Material Systems Analysis Activity)/Duane Model
- Endless-Burn-In Model
- Modified Duane Model
- Cox-Lewis Model
- Time Series Modeling.

A data base was compiled from development testing and/or field service for six major components, representative of both electronic and mechanical control systems. Two components, the F100 JFC-90 Electronic Engine Control (EEC) and F100 MFP-330 Main Fuel Pump System, were selected for use as a basis for comparing each model's ability to forecast reliability over the short-and long-term.

The models were then compared via the evaluation criteria, and the AMSAA/Duane Model was selected as the best model. The Time Series and Endless-Burn-In models were also considered acceptable. The best method for estimating the AMSAA model parameters which must be done prior to model application was found to be the method of maximum likelihood.

The AMSAA/Duane Model and Box-Jenkins Time Series Modeling were reviewed in detail to determine how to use confidence intervals for estimates of reliability and goodness-of-fit tests to quantify the errors in assessment and forecasting of reliability. Model assumptions, strengths and weaknesses were also reviewed and documented.

A procedure was then drafted to assist the user in applying reliability growth modeling techniques to advanced control system development programs. Two EEC's, the P&W Military F100 JFC-90 and the P&W commercial JT9D EEC 103-2, were used as examples in the procedure.

Assessment of the applicability of growth modeling to warranty programs revealed that care should be taken when using the models to assign warranties because of their inadequacies in making long-term projections. However, reliability-growth modeling does appear to provide valuable guidance in assessing the impact of warranty programs over the near term.

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SECTION II

RELIABILITY GROWTH MODEL RESEARCH AND SELECTION

A. TASK 100 — LITERATURE SEARCH AND MODEL SELECTION

1. Literature Search

The United Technologies' library svstem was used in conjunction with several area university libraries (University of Florida, Florida State University and Florida Atlantic University), a computer search program (DIALOG) and communication with several specialists to compile a bibliography of more than 200 books, articles and reports related to reliability-growth modeling. This bibliography, shown here as Appendix A, contains literature which covers the past 30 years of work on reliability-growth assessment, prediction and control. This literature has been grouped by model type and subject matter for easy reference as shown in Table 1.

TABLE 1. LITERATURE FROM APPENDIX A CATEGORIZED BY MODEL TYPE AND CONTENT

	Subject Matter				
Model Type	*Theory*	*Methods*	*Electronic Application*	*Mechanical Application*	*General Discussion*
Deterministic	1,15,16, 17,18,19, 27,34,36, 41,54,55, 56,58,64, 73,84,115 145,172, 177,178, 195,200, 204,208, 210,212	1,9,15,18 23,24,25, 27,32,36, 40,41,53, 64,68,73, 76,145, 177,178, 189,195, 205,208, 209,210	23,24,25,38, 46,133,142, 153,174	9,133,177, 190,209	4,16,18,21, 23,24,25,32, 34,37,40,41, 42,45,46,53, 57,62,63,65, 70,71,81,82, 102,115,142, 145,162,163, 174,182,183, 185,197,203, 204,209,214
Renewal Process	14,18,29, 30,44,59, 175,176	29,30,41, 59,175, 176			44
Markov Process	2,3,14, 18,26,34, 44,59,66, 175,176, 184,188	2,14,26, 78,175, 176		2,78	14,34,44
Nonhomogeneous Poisson Process	8,11,14, 16,18,44, 48,49,50, 51,52,61, 71,72,75, 77,124, 145,149, 164,195, 196,211	11,12,13, 14,48,49, 50,51,52, 61,71,72, 75,77,124 145,149, 195,196	133	11,12,13,48, 49,61,133	8,10,11,13, 16,44,45,48, 60,67,145, 186,197
Semi-Markov Process	167,175, 176,189	9,167,175 176,189		9	
Time Series Process	3,5,33, 74,85	5,33,39, 74,83,191 192,193		191,192,193	5,31,33
Bayesian	69,83,88, 104,105, 134,165, 194	69,83,88, 134,165, 194			

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a. Dialogue With Specialists

A personal dialogue was established with three specialists in reliability-growth modeling as follows:

Mr. Alexander G. Bezat
Honeywell, Inc. Avionics Div.
1625 Zarthan Ave.
St. Louis Park, MN 55416

Dr. Larry Crow
US Army Material System — Analysis Activity
Aberdeen Proving Ground
Aberdeen, MD 21005

Dr. Ronald Scott
United Technologies Inc.
West Palm Beach, FL 33402.

Mr. Alexander Bezat, a reliability-growth modeling specialist at Honeywell Inc., has successfully applied the Endless-Burn-In (EBI) model to commercial aircraft avionic controls' programs. Mr. Bezat was very helpful in clarifying the assumptions associated with the EBI model and how it has been applied at Honeywell. He also contributed several published and unpublished papers on application of the EBI model as noted in Appendix A.

Dr. Larry Crow, a reliability-growth modeling specialist with the Army Material Systems Analysis Agency (AMSAA) has developed and successfully applied the AMSAA/Duane modeling procedure and was responsible for the development of MIL-HDBK-189 (Appendix A), the most inclusive and widely accepted document on reliability-growth modeling presently available. Dr. Crow was helpful in clarifying the underlying assumptions of the AMSAA model and how to apply it to a wide range of data. He also recommended methods for estimating the influences of delayed fixes on reliability. He also made available several papers on reliability-growth.

Dr. Ronald Scott is a mathematician with United Technologies Research Center and a specialist in Kalman Filtering and Markov Processes. Dr. Scott was helpful in clarifying the requirements for applying Markov Process models. A dialogue was also established with several other specialists in the field of reliability or related fields, as listed in Table 2.

b. Established Criteria

A set of objectives and qualitative criteria was established to judge the models' effectiveness. This criteria, as listed in Appendix B, was primarily concerned with flexibility, ease of use, short- and long-term forecasting precision, and statistical validity. A list of statements describing the criteria is also presented in Appendix B. These statements were necessary so that qualitative measures of goodness could be assigned.

2. Models Reviewed and Selected

The models reviewed fell into one or more of the following general categories: Deterministic, Poisson Process, Markov Process/Time Series, Bayesian Models.

TABLE 2. DIALOGUE

DISCUSSIONS WITH SPECIALISTS

1	Mr. H. E. Ascher	Naval Research Lab, Wash., DC	NHPP/Semi Markov Process
*2	Mr. A. Bezat	Honeywell Inc., Minn., MI	Deterministic (Endless-Burn-In)
*3	Dr. L. H. Crow	U.S. Army Material Systems, Aberdeen, MD	NHPP (AMSAA Model)
4	Dr. K. W. Fertig	Rockwell Science Center, CA	General Comments
5	Dr. J. M. Finkelstein	Hughes, Inc., CA	NHPP (Modified Duane Model)
6	Dr. P. Holmes	Clemson University, SC	Markov Processes/NHPP
7	Dr. W. Jewell	University of California, Berkeley, CA	Bayesian Models
8	Dr. P. A. W. Lewis	Post Graduate Naval School, Monterey, CA	Time Series
9	Dr. W. A. Meeker	Iowa State University, IA	General Comments
10	Dr. W. Nelson	General Electric, Schenectady, NY	General Comments
11	Dr. K. Portier	University of Florida, FL	Semi-Markov Processes
12	Dr. F. Proschan	Florida State University, FL	General Comments
13	Dr. R. Scheaffer	University of Florida, FL	Semi-Markov Comments
*14	Dr. R. Scott	United Technologies Corp., W. Palm Beach, FL	Markov Processes
15	Dr. N. D Singpurwalla	George Washington University, Wash., DC	Time Series

*Indicates personal visits.

a. Deterministic Models

The form of Deterministic models (models which contain no random elements) is assumed to be known. That is, it is known that reliability grows via the Deterministic model. The parameters are estimated and reliability is then calculated from the model. Examples of Deterministic models reviewed in Table I are as follows:

- The Duane Model:

$$\text{Cumulative MTBF} = \gamma\,(\text{Cumulative Time})^{\alpha}$$

- The Endless-Burn-In Model:

$$\text{Instantaneous failure rate} = K\,(\text{Average Age})^{-\alpha} + \lambda_R$$

- The Gompertz Model:

$$\text{Cumulative failure rate} = K \cdot A^{B \cdot (\text{Cumulative Time})}$$

- The Grumman Model

$$\text{Instantaneous failure rate} = A\,e^{-K\,(\text{Cumulative Time})} + B\,.$$

b. Poisson Process Models

This class of models does not assume that the form of the reliability-growth function is known, but only that it can be approximated statistically. Since the model form is only an approximation, the quality of fit tests are necessary to determine if the approximation is reasonable. The Poisson Process model assumes that events occur based on a poisson type of distribution. That is, the probability of realizing K events (failures) by some test time t is as follows:

$$P(x = k) = \frac{[M(t)]^k\,e^{-M(t)}}{k!}$$

Here $M(t)$ is the mean value function. If $M(t) = \lambda t$, the process is a Homogeneous Poisson Process (HPP) and the time between failures follow an exponential distribution (the probability of failure by time $t = 1 - e^{-\lambda t}$).

The Poisson Process also assumes independent increments. Renewal Theory generalizes the HPP by allowing time between failures to have other distributions besides the exponential. A renewal process is a sequence of random variables $[Y_1, Y_2, \ldots]$ of the form $Y_i = X_1 + X_2 + \ldots + X_i$, where each X_i comes from a common distribution $F(X)$. Therefore, the renewal model assumes that each repair returns the system to good-as-new state. If the Poisson Process has a *more general* mean value function than $M(t) = \lambda t$, it is said to be a Nonhomogeneous Poisson Process (NHPP). The intensity function, $\rho(t)$, of a Poisson Process is the rate at which failures are occurring and is related to the mean value function as follows:

$$M(t) = \int_0^t \rho(x)\,dx.$$

The poisson process can be generalized further (Thompson) to a *general point process* but little applied work has been done at that level of abstraction. The NHPP model is the most popular since it can model a system that is wearing out. Several intensity functions have been used with the NHPP model. The following are examples:

- The AMSAA model used by Dr. Crow,

  $\rho(t) = \lambda\beta$ (cumulative test time) $^{\beta-1}$;

- The modified Duane Model used by Dr. Finkelstein,

  $\rho(t) = \lambda\beta$ (cumulative test time)$^{\beta-1} + \theta$;

- The Cox-Lewis model,

  $\rho(t) = e^{\alpha + \gamma\,(\text{cumulative test time})}$.

c. *Markov-Processes/Time Series Models*

H. Akaike (Appendix A) showed that Markovian processes and the autoregressive moving average process were related. A Markov process is a process that moves from state i to state j with some probability, P_{ij}. This probability is independent of all past states, and is dependent only on the present state. The time spent in each state is an exponentially distributed random variable. If the time in residence is not exponential, the process is said to be a semi-Markov process. Markovian processes usually require more data, to estimate the probabilities, than is generally available.

Dr. Singpurwalla (Appendix A) suggests that time series methods developed by Box and Jenkins be used to model reliability growth. The Box-Jenkins Autoregressive Integrated Moving Average (ARIMA) modeling approach is attractive in that no specific model need be selected in advance. The models are flexible in that they can be applied to a wide range of data and the methodology has a built-in theory of forecasting. An ARIMA first difference model could be written as follows:

$\lambda_{c,T} = \lambda_{c,T-1} + \varepsilon_T$, where $\lambda_{c,T}$ is the cumulative failure rate at time T and ε_T is the random (normal (O, σ)) error at time T.

B. SUMMARY OF TASK 100

During Task 100, the literature was reviewed (Appendix A), the criteria established (Appendix B) and five models were recommended for carryover into Task 200. These models were:

(1) The AMSAA/Duane Model (NHPP)
(2) The Modified Duane Model (NHPP)
(3) The Cox-Lewis Model (NHPP)
(4) The Endless-Burn-In Model (Deterministic)
(5) Box-Jenkins ARIMA Models (Time Series Process).

These models were representative of three of the four classes of models considered. Since little information was available on Bayesian models, they were not recommended for further review.

SECTION III

EVALUATION OF RELIABILITY GROWTH MODELS AND THE SELECTION OF THE BEST MODEL(S)

A. DATA COLLECTION AND REVIEW

1. Selected Components

The two major controls' components selected for use in this study were the F100 Electronic Engine Control (EEC) and the F100 Main Fuel Pump Systems (MFPS) shown in Figures 1 and 2. The F100 EEC is an engine-mounted, digital, electronic supervisory control. The F100 MFPS is a high technology vane pump and variable displacement controller. F100 components currently have over 3 million hours of operational experience in the F-15 and F-16 aircraft. These components were selected because of the quantity and quality of data available and because they represent the generic types of components that are applicable to future engine control systems.

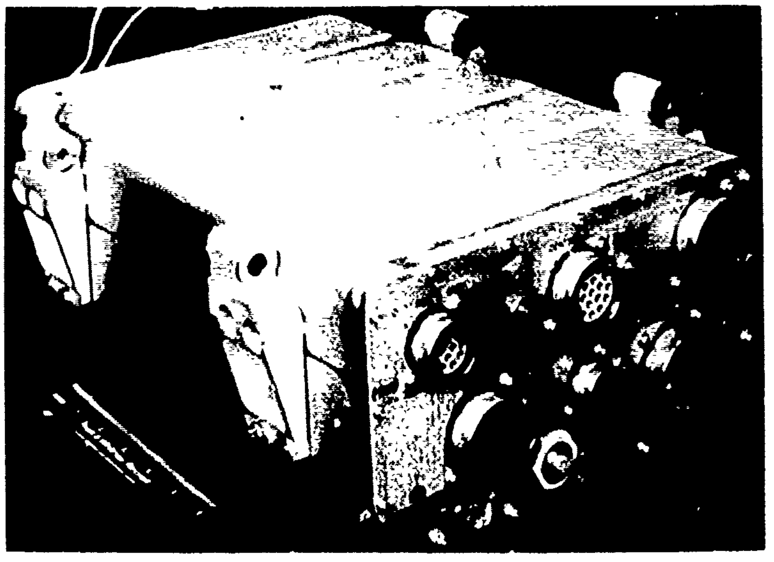

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Figure 1. F100 Electronic Engine Control

In order to expand the number of data bases and number of different type components available for model evaluation, two additional F100 components were added: These were the Compressor Inlet Variable Vane (CIVV) Actuator and the TT2.5 Sensor shown in Figures 3 and 4. These four F100 controls components were used throughout the study for evaluating and selecting the reliability growth model.

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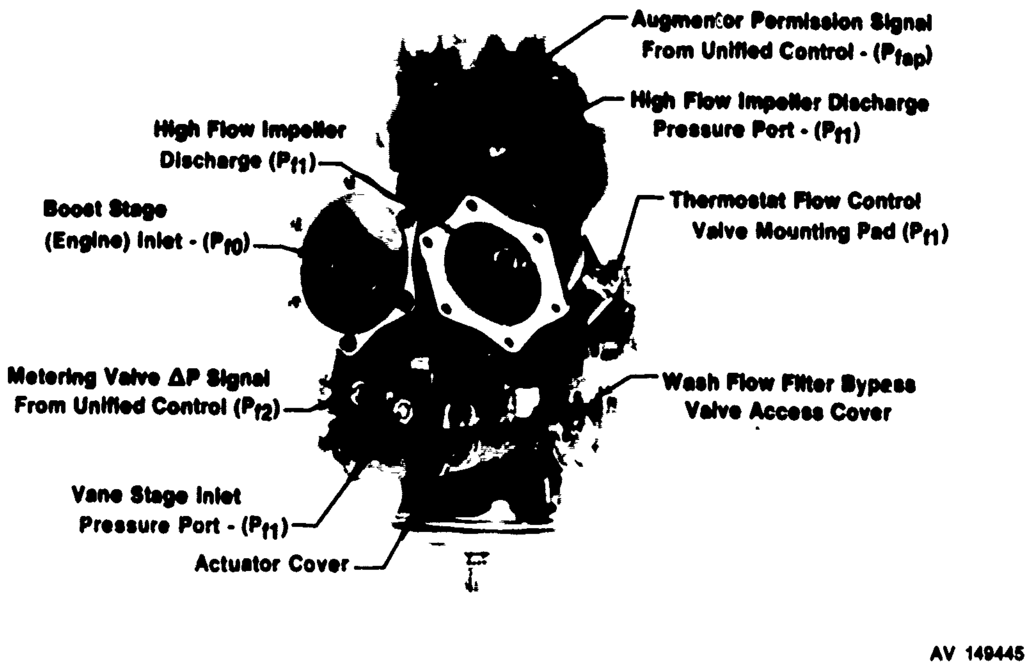

Figure 2. F100 Main Fuel Pump

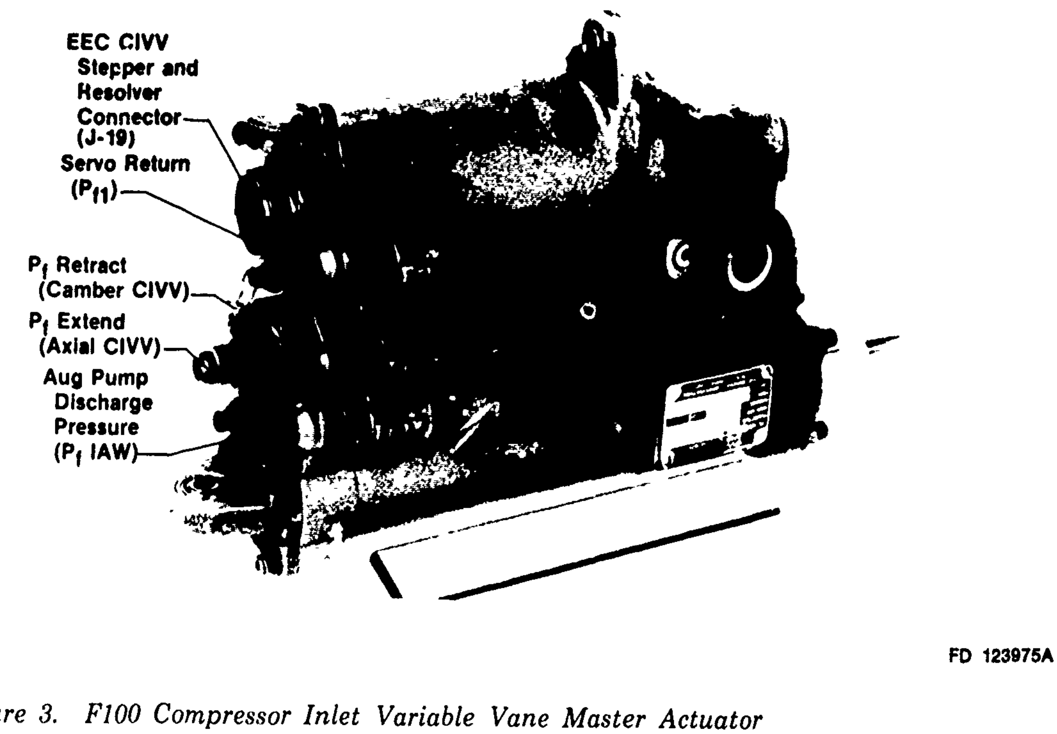

Figure 3. F100 Compressor Inlet Variable Vane Master Actuator

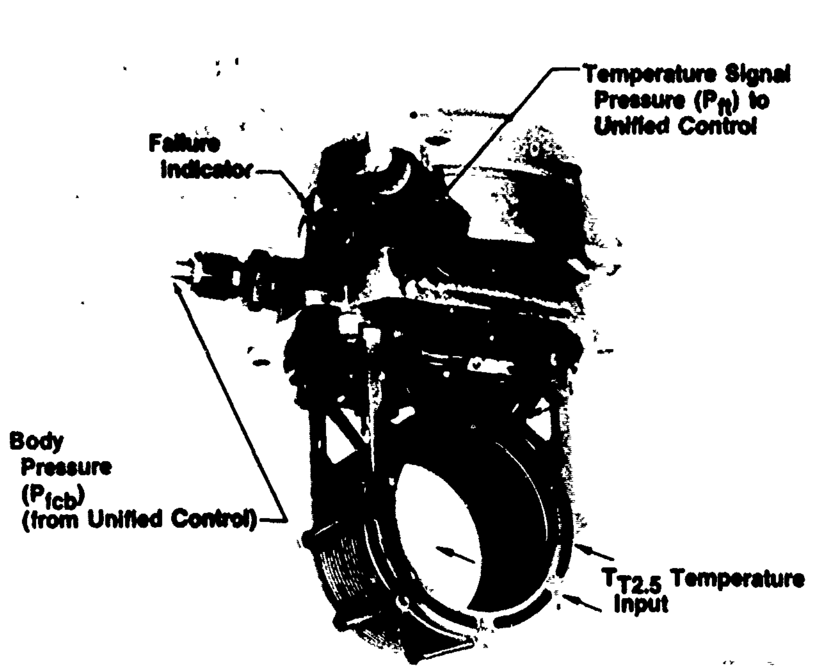

Figure 4. F100 Fan Exit Temperature (TT2.5) Sensor

Two additional electronic engine controls currently in development were also included in the data base for model evaluation. The Digital Electronic Engine Control (DEEC) developed for the F100 engine has completed 50,000 hours of Combined Environment Reliability Testing (CERT). CERT is an effective method for subjecting a component to the combined environment (vibration, temperature, pressure, humidity) that the component would experience during operatiional service. Testing under these controlled conditions, 24-hours per day, permits the accumulation of many hours of exposure during a much shorter time period than could be accomplished during operational service. The Electronic Engine Control (EEC) for the commercial JT9D engine has also completed a 50,000-hour CERT and has accumulated over 300,000 hours of flight test experience on JT8D engines. These programs represent current state-of-the-art electronic controls with testing designed specifically to find and eliminate problems.

2. Data Collection

a. F100 Controls Development Data

Development data for the F100 controls' components was extracted from the P&W/GPD reporting system. Shown in Table 3 is an example of the data available from this computerized system.

This data system includes malfunctions from engine development, flight test, and production testing of F100 components covering the time period 1969 to 1982. All data for the four F100 controls' components was reviewed to ensure that all reported malfunctions were chargeable to the component and not a result of test stand interface or instrumentation problems.

TABLE 3. F100 REPORTING DATA

FMR No.: *DATE: 75/12/19* *ENGINE: P0264* *TYPE: JTF22* *MODEL: F100(3)* *RUN NO.:* *BUILD: B* *BASE: F9096*

APE: COMPONENT: CONTROL-EEC PART NO.: 4044335 SERIAL NO.: AESC0297 OCCURRED AT: FAIL TYPE:

DESCRIPTION:
NO CIVV SCHEDULE

ANALYSIS:
INTERMITTENT SOLDER JOINT AT TB303-12 BETWEEN FLEXTAPE AND PIN.

CORRECTIVE ACTION:
ASSEMBLY PROCESS MODIFIED TO REQUIRE PREFOLDING OF ALL INTERBOARD FLEXTAPES PRIOR TO INSTALLATION. THIS PROVIDES STRESS RELIEF TO REDUCE FORCES ON SOLDER JOINTS.

b. F100 Field Data

Field data for the F100 components was taken from the P&W/GPD Critical Components Reliability Assessment (CCRA) system. Shown in Table 4 is a sample of the data available from this system

This data system currently contains over 7000 removal records for the EEC, 2300 for the MFP, 1000 for the CIVV Actuator, and 1900 for the TT2.5 sensor.

The CCRA system uses data from multiple sources as shown in Figure 5. The percentages shown indicate how much each data source contributes to the resulting CCRA data base.

c. Advanced Controls Development Data

Data on the CERT for the F100 DEEC and JT9D EEC was taken from final reports by the component manufacturer. These reports contain detailed information on each failure, including information on corrective action implementation. The testing was conducted for the purpose of growing reliability and therefore forms an important data base for model evaluation. Table 5 shows an example of the data summary available from this testing.

Data from the EEC flight testing on JT8D engines in Boeing 727 aircraft was obtained from P&W Commercial Engineering. Complete information on each malfunction, analysis, and corrective action was included.

d. Operating Hours

Operating time for the F100 components was taken from F100 development history, flight test documentation, and USAF operational flight hours records. Where only flight hours were available, they were converted to operating hours using well-documented historical conversion factors. All operating time for F100 data was grouped, i.e., the fleet time was reported in monthly increments. Operating time from the advance controls testing was taken from detailed test reports and is continuous data, i.e., fleet time at each failure.

TABLE 4. CCRA DATA EXAMPLE

Serial Number	*Engine Number*	*Tot Hour*	*EFH*	*Vendor Part Number*	*Data Source* *66-1*	*Vendor*	*ILS*	*EOA*	*SA-ALC*	*Prod*	*Other*	*Oper No.*	*Hard No.*	*Stat*	*Disposition*	
	Base													Incident	Vend Rpt No.	
AESC 1057	680773	933	0	773098011		020482	012182				012182	165	516	Conf	Rts 03/16/82	
	KADENA OKI			Operation Problem	EEC will not trim CIVV's											V82392
				Findings	Replaced Z301 Replaced VR107									U O O P		
AESC 1320	680550	1241	0	773098012		051982		051482	051982		051482	75	516	Conf	Rts 06/17/82	
	SA-ALC			Operation Problem	High FTIT and RPM at intermediate											V82643
				Findings	Replaced VR 309, Replaced Cover									0 0 0 0		

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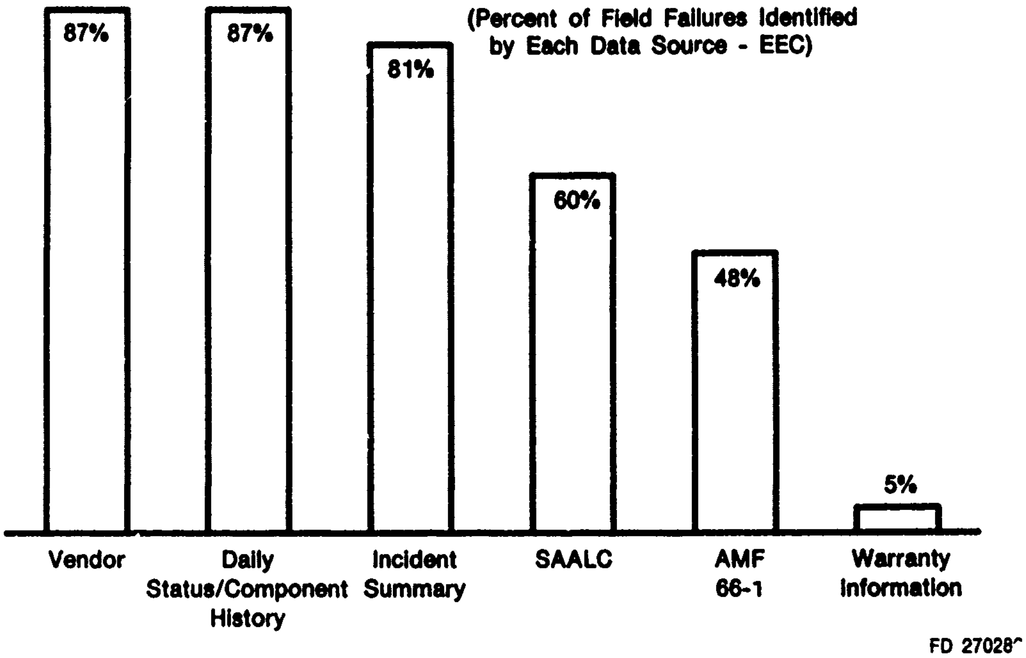

Figure 5. CCRA Data Sources

TABLE 5. CERT SUMMARY DATA EXAMPLE

Item No	*MR No.*	*Unit No*	*Fault Date*	*UUT Time to Malfunction*	*All UUT Time to Malfunction*	*Malfunction Cause*	*Failure Classification*	*Corrective Action*
1	E001	6	01/07/81	2.0	4.0	Socket Damage	EMW	Add HS9575 Req'ts
2	E002	3	01/07/81	10.4	20.8	TPT2.Interm.Unk.	Other	Monitor and TBD
3	E003	2	01/13/81	21.9	295.0	Socket Damage	EMW	Add HS9575 Req'ts
4	E004	5	01/13/81	22.5	295.0	Socket Damage	EMW	Add HS9575 Req'ts
5	E005	4	02/17/81	628.0	2459.0	Socket Damage	EMW	Add HS9575 Req'ts
6	E006	3	03/04/81	1070.0	3100.0	Socket Damage	EMW	Add HS9575 Req'ts
7	E008	6	05/12/81	1351.1	7274.0	Incorrect tool use	EMW	Added info + QC test
8	E009	2	06/08/81	1735.7	8996.3	Socket/lead contact	EMD	Process change
9	E012	4	08/18/81	3466.0	15,522.1	Socket/lead contact	EMD	Process change
10	E013	3	09/29/81	4651.0	19,988.7	Lead abrasion	EMD	In process, TBD

3. Review and Classification of Failure Data

All reported malfunctions in the data base were reviewed and classified using the following criteria:

- Confirmed — The malfunction was confirmed to be a component malfunction requiring repair and/or corrective action.
- Induced — The malfunction was induced by the engine, other controls components, test stand operation, mishandling, or misassembly.
- Unconfirmed — The malfunction did not result in an identified component problem and required no component repair.

- Logic — The malfunction was due to a logic problem which required changes to the program memory (PROM) to correct schedules, fault accommodation, etc.

For the purposes of reliability-growth model assessment, only confirmed failures were used. Failures other than confirmed are not necessarily a function of the true component reliability and were therefore eliminated from the analysis. An example of the resulting data base used for all further analysis is shown in Table 6.

TABLE 6. DATA LISTING FOR GROWTH MODEL INPUT; EXAMPLE — EEC

Serial No.	*Date*	*Hours*	*Status*	*Location*
AESC 0113	741201		Confirmed	Unknown
AESC 0206	760520	00168	Confirmed	Luke
AESC 0503	770721	00308	Confirmed	Langley
AESC 1393	801220	00527	Confirmed	McDill
AESC 0368	820225	00463	Confirmed	McDill

This data base includes the following number of failures for each listed component of the F100 control system covering development, production, flight test, and field data from 1969 through 1982:

Component	*No. of Failures*
EEC	2202
MFP	415
CIVV CTL	884
TT2.5 Sensor	293

Data for the F100 EEC was further analyzed for use in the Endless-Burn-In (EBI) model evaluation. This model is limited to failures of solid-state electronics. Those failures, due to solid-state electronics, were identified and extracted from the F100 EEC data. In order to expand the data base for EBI assessing, field data for the F100 Engine History Recorder (EHR) was also reviewed and used for evaluation.

4. Subcontractor Participation

As a part of this program, both Hamilton Standard, F100 EEC supplier, and Chandler Evans, F100 MFPS supplier, participated in reviewing the failure data and reliability-growth models being evaluated. Each supplier was provided with all failure data on their component. The suppliers then reviewed that data and provided revisions and corrections where required. All comments were incorporated into the data bases for the EEC and MFPS.

In addition to data review, these suppliers reviewed the growth models and work-to-date on the program.

5. Initial Evaluation of Data

The initial evaluation of failure data involved plotting the data as MTBF versus time and reviewing the resulting trends. Questions which were being investigated include: (1) do hardware changes explain trends, (2) are other influences such as operational usage or maintenance practices influencing the data, and (3) where do different program phases start and stop. Figure 6 shows the results of this evaluation for the F100 EEC.

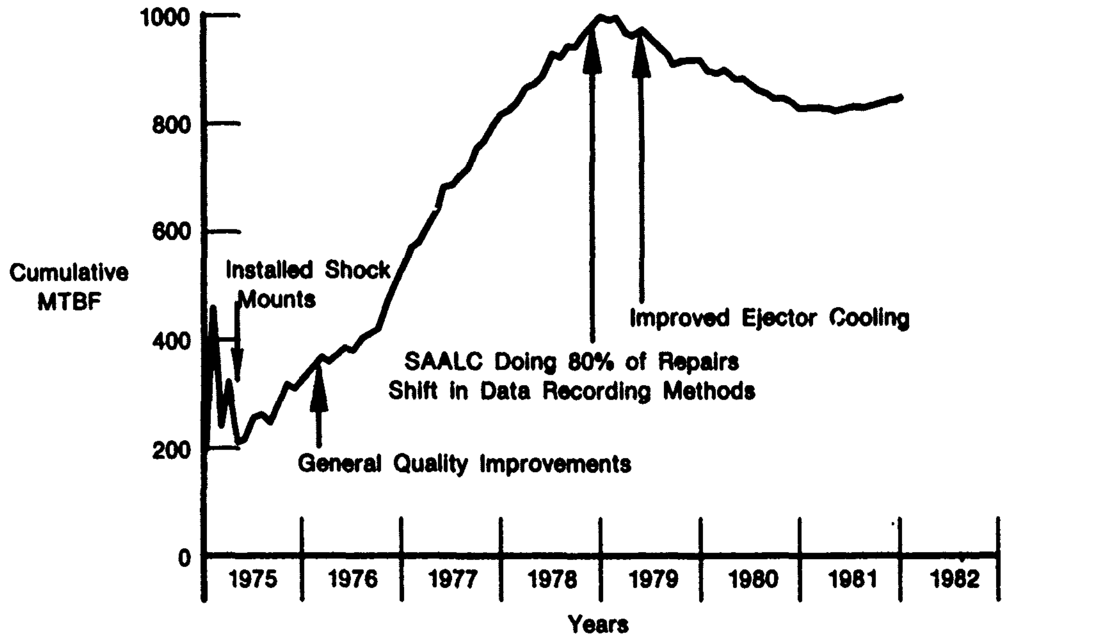

Figure 6. EEC Initial Evaluation

Major design changes to the EEC, changes in environment and changes in maintenance personnel, are indicated on this plot. Correlation between design changes and reliability growth were evident in mid-1975 when shock mounts were installed on the EEC. In mid-1979, the cooling environment was improved with no immediate shift in the growth curve. This is thought to be due to the amount of inertia (large number of units, hours, and failures) in the data system and the slow retrofit program used to install the change (still incomplete in mid-1983).

A second evaluation was made to investigate the *check mark* theory. This theory states that following introduction to service, the reliability of a component drops off sharply and then recovers to near the original growth trend. To evaluate this phenomenon, failure rate versus time plots for the four F100 components were made as shown in Figures 7 and 8. Introduction to service for these components occurred in late 1974. A check mark on these plots would be seen as an upward spike near this point in time. As can be seen from these plots, no real evidence of a check mark exists in the data.

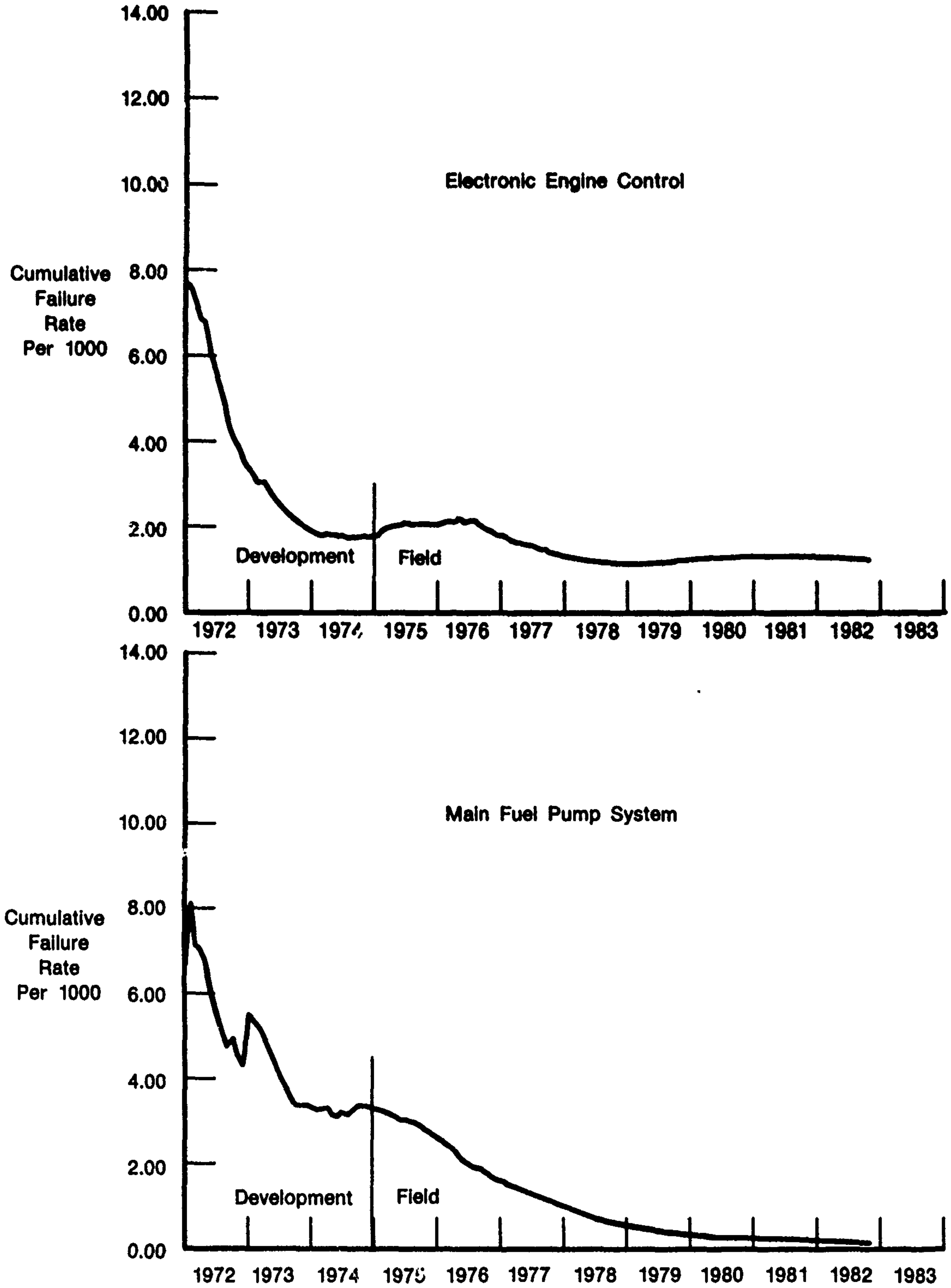

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Figure 7. GAPCEEC Cumulative Failure Rate Analysis — EEC (top photo) and MFP System (bottom photo)

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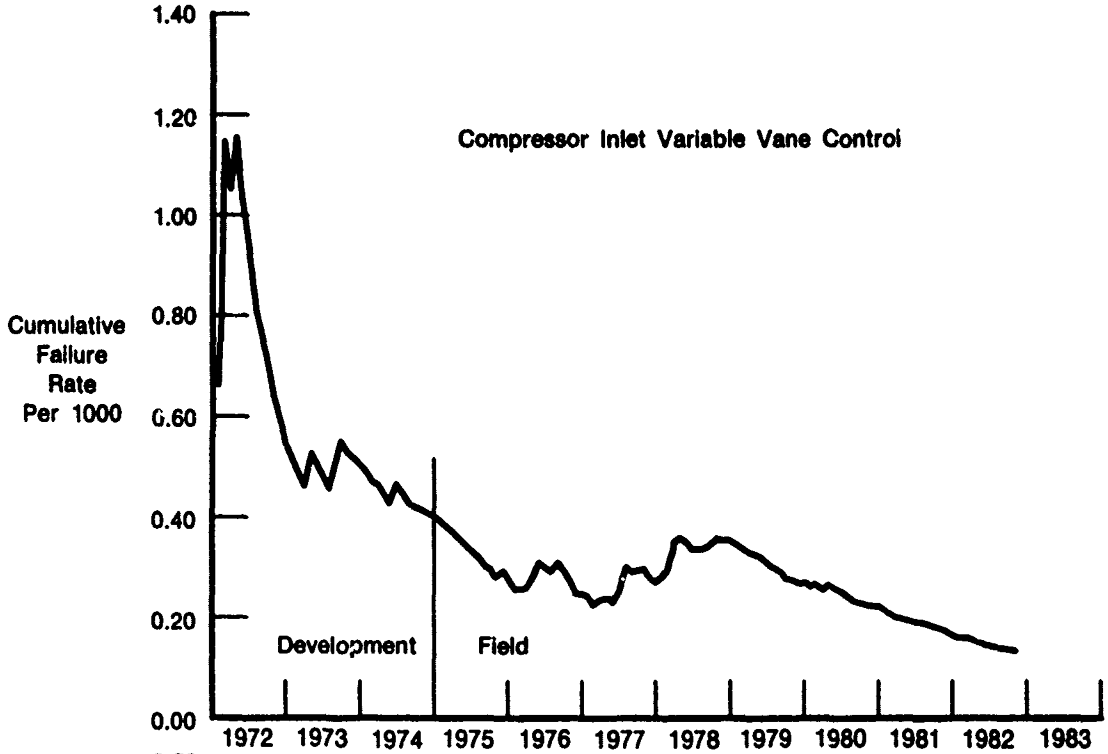

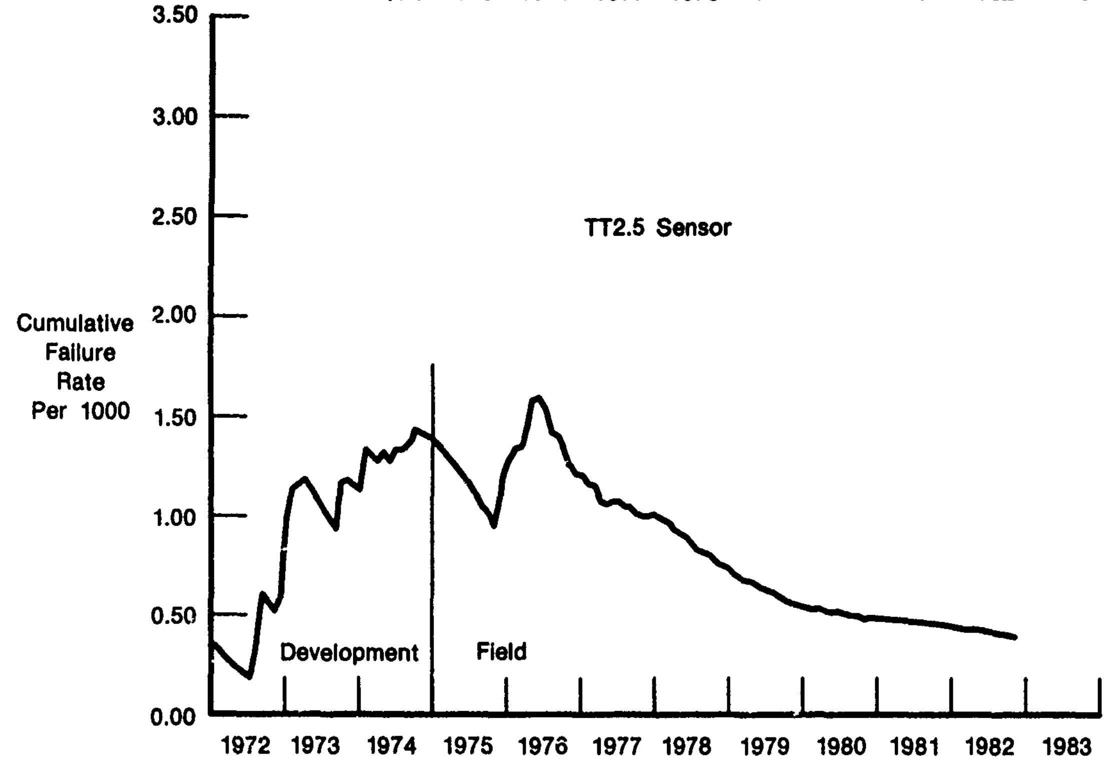

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Figure 8. GAPCEEC Cumulative Failure Rate Analysis — CIVV Actuator (Top Photo) and TT2.5 Sensor (Bottom Photo)

B. EVALUATION OF THE MODELS

The primary objective of this part of the study was to evaluate the five models (AMSAA, Modified Duane, Cox-Lewis, EBI, and Box-Jenkins Time Series) in a comparative manner, relative to the criteria established in the research and selection phase (Appendix B) and determine which models are the most appropriate for use in modeling the reliability growth of electronic engine controls.

1. Time Series Analysis Study

a. *Objectives/Background*

The objective of this part of the study was to assess the usefulness of ARIMA (Autoregressive Integrated Moving Average) models in forecasting controls reliability. The first five criteria involve the models ability to accurately forecast reliability at some future point in time. For the deterministic (EBI) model and the NHPP (AMSAA, Modified Duane and Cox-Lewis) models, the form of the model and the reliability parameter (MTBF, failure rate etc.) were prespecified. For Box-Jenkins Time Series modeling a separate study was required to determine the form of the model and the appropriate reliability parameter to be modeled.

b. *The Data Used in this Analysis*

The data consisted of incident counts and total operating time by month for each of the four main components (EEC, MFP, CIVV Actuator, and TT2.5 sensor) operating in development, production, flight and field environment. In addition the digital electronic engine control (DEEC) data from combined environment reliability test (CERT) and the engine history recorder (EHR) field data were used, as shown in Table 7.

TABLE 7. DATA SETS AVAILABLE FOR ANALYSIS

	Development	*Production*	*Field*	*Combined*
EEC	X	X	X	X
EEC (solid state only)	X	X	X	X
Main Fuel Pump	X	X	X	X
CIVV Actuator	X	X	X	X
TT2.5 Sensor	X	X	X	X
Engine History Recorder			X	
DEEC	X			

Also, EEC Flight Test Data

c. *Methodology*

For determining the *best* time series model and the most appropriate reliability parameter to be used in time series modeling, the following variables were considered as possible candidates for use as a measure of reliability:

1. Cumulative failure rate (λ_c) = cumulative incidents/cumulative operating time.

2. Cumulative mean time between failures ($MTBF_c$) = $1/\lambda_c$.

3. Instantaneous failure rate (λ_i) = incidents for the month/operating time for the month.

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4. Instantaneous mean time between failure ($MTBF_i$) = $1/\lambda_i$.

5. Cumulative failures.

6. Instantaneous failures by month.

While it was unlikely that the last two variables would be serious candidates, they were included to verify the need for a time factor.

The four ARIMA models reviewed in this study are as follows:

1. ARIMA (0,1,0) A first difference model.
2. ARIMA (0,1,1) A first difference moving average model.
3. ARIMA (1,1,0) A first difference autoregressive model.
4. ARIMA (1,1,1) A first difference autoregressive moving average model.

These terms are discussed further in Appendix E.

For each variable, model, and data set the model is fit to the first 18 months of data (18 months after the first failure), a forecast of the reliability is made for the next 12 months, and summary statistics are computed. Next the model is fit to 18 plus 6 months worth of data and the process is repeated. This methodology is graphically illustrated in Figure 9. The summary statistics used for evaluating the forecasts are as follows:

- Root Mean Square Error (RMSE)

$$RMSE = \left[\frac{\Sigma\,(\text{Predicted Reliability} - \text{Observed Reliability})^2}{12\sim(\text{months})}\right]^{1/2}$$

- Largest Absolute Error = Max (Predicted Reliability — Observed Reliability)

- Mean Delta Percent Error

$$= \frac{\Sigma\,(\text{Predicted Reliability} - \text{Observed Reliability})}{12\ (\text{months})} *100$$

- Mean Absolute Error

$$= \frac{\Sigma\ \text{Absolute Value}\,(\text{Predicted Reliability} - \text{Observed Reliability})}{12\ (\text{months})}$$

A preliminary analysis indicated that the summary statistics were highly correlated (95%), therefore only the RMSE was analyzed in detail. The RMSE was selected because of its popularity in the literature and because it is the most stable.

Multivariate cluster analysis techniques were initially used to analyze these RMSE statistics, and identify the best model variable combination. The extreme overlap of clusters rendered a choice of the best group impossible, thus indicating the need for a finer analytical approach.

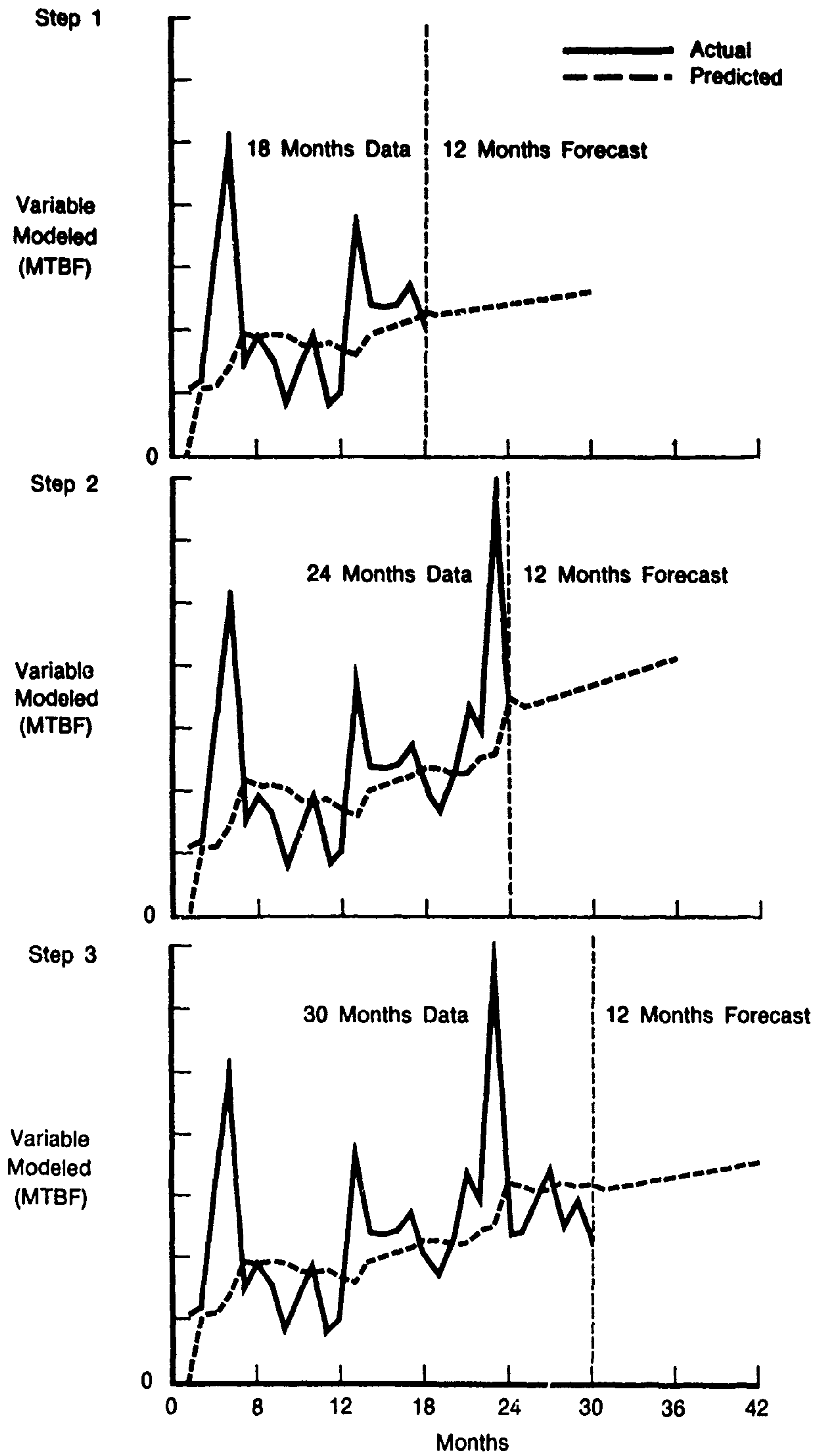

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Figure 9. Methodology for Computation of Errors

The most detailed and adaptable statistical technique available for analysis of experimental data is the Analysis of Variance. Three important assumptions in an Analysis of Variance Study are that the errors are from a normal distribution, have a common variance, and are independent of each other. Normality and homogeneity of variance were approximated after performing a log transformation of the data. The 24 variable/model (6×4) combinations were made comparable by standardizing each cell to a population mean goal of zero error (i.e., X (observation - 0 goal)/SD_{cell}). This converted all data to dimensionless standard deviations from a goal of zero and made it amenable to various combinational comparisons.

In some cases (variable/model/data base/size) parameter estimates failed to converge or converged to obviously wrong points and were classified as solution failures. The number of solution failures for each variable/model combination was used to compute a weight variable for each cell of the form.

Wt = 1.0 − No. Solution Failures/(No. Solution Failures + No. Solutions).

Because of the missing data (i.e., solution failures), the experiment was classified as an unbalanced design or an analysis of variance (ANOVA) model of less than full rank. Ultimately, an unbalanced, weighted, two-way (variable/model) ANOVA was solved to obtain the desired analytical comparisons.

d. Model Performance

Table 8 shows the general ANOVA table for this experiment. The ANOVA model, which is a linear model describing all the variable/model combinations, accounts for most of the variation present in the RMSE points (R-Square = 0.942). Examination of the residuals of the ANOVA model fit indicated that the basic assumptions (normality, homogeneity of variance, and independence) were reasonable.

TABLE 8. OVERALL ANOVA

Source	*DF*	*Sum of Squares*	*Mean Square*	*F Value*	*Pr F*
ANOVA Model	23	132,160.068	5746.099	5590.02	0.0001
Error	7871	8090.747	1.028		
Corrected Total	7894	140,250.815			
R-square:	0.942				

Initial analysis of the ANOVA model indicate that there is a significant difference between some or all of the six variables tested (P = 0.0001), there is a significant difference between some of the variable/model combinations (P = 0.0238), and no significant difference between the ARIMA models (P = 0.1557) as summarized in Table 9. (Note: P should be less than 0.1 to detect a 90% significance level.)

TABLE 9. DETAILED ANOVA

Source	*DF*	*Type III SS**	*F Value*	*Pr F*
Test Variable	5	132,126.228	25,707.53	0.0001
ARIMA Model	3	5.349	1.73	0.1557
Variable × Model Interaction	15	28.457	1.85	0.0238

* Type III Sum of Squares is specifically designed to handle ANOVA analyses with missing data.

Using Least Squares (LS) Means (a method whereby missing data — solution failures — are compensated for by projecting what a cell mean would be if there were no missing data) and testing the null hypothesis of no differences between cell means, the variable Cumulative Failures/Cumulative Engine Flight hours (cumfail/CEFH or λ_c) was found to generate the least amount of error over all data bases, ARIMA models and data base sizes. The LS means are shown in Table 10. Each LS mean is significantly different from each of the others. Recall that these values are in terms of the number of standard deviations away from a goal of zero.

TABLE 10. LEAST SQUARE MEANS FOR VARIABLES

Test Variable	*Least Squares Mean*
Cumfail/CEFH	0.000002108
Failures	0.000065415
Failures/EFH	1.47617
Cumfail	96.3180
1/(Cumfail/CEFH)	8253.4604
1/(Failures/EFH)	242,541.4670

The λ_c variable satisfies one of the assumptions of ARIMA modeling: That time intervals are equal. By using rates as input to ARIMA modeling, this interval equivalence is obtained.

In order to discern which λ_c/ARIMA model combinations were generating significant interactions, orthogonal comparisons or contrasts were computed and listed in Table 11. Using the ANOVA contrast results for the summary statistics Largest Absolute Error and Mean Absolute Error in conjunction with those for RMSE, the clearest path through all the overlapping significant and nonsignificant differences appears to be the following:

- λ_c ARIMA (0,1,0) equivalent to λ_c ARIMA (0,1,1)
- λ_c ARIMA (1,1,0) equivalent to λ_c ARIMA (1,1,1)
- λ_c ARIMA (0,1,0) and λ_c ARIMA (0,1,1) significantly different from

  λ_c ARIMA (1,1,0) and λ_c ARIMA (1,1,1)

TABLE 11. CONTRASTS OF ARIMA MODELS

Contrast (for λ_c)	*DF*	*SS*	*F Value*	*Pr F*
0,1,0 vs others	1	3.003	2.92	0.0874
0,1,0 vs 0,1,1	1	0.194	0.19	0.6641
0,1,0 vs 1,1,0	1	4.051	3.94	0.0472
0,1,0 vs 1,1,1	1	3.201	3.11	0.0777
0,1,1 vs 1,1,0	1	2.473	2.41	0.1[illegible]9
0,1,1 vs 1,1,1	1	1.822	1.77	0.1831
0,1,0γ0,1,1 vs 1,1,0	1	4.283	4.17	0.0413
0,1,0γ0,1,1 vs 1,1,0 1,1,1	1	5.651	5.50	0.0191

As to which cumfail/CEFH:ARIMA model combination is best regarding generation of the least amount of error in a 12-month forecast, Table 12 shows that ARIMA (1,1,0) and ARIMA (1,1,1) are mildly better than ARIMA (0,1,0) and ARIMA (0,1,1). On the other hand, comparison of the LS means shown in Table 13 indicates that overlap is such that there may not be a best answer.

TABLE 12. LEAST SQUARE MEANS FOR λ_c ARIMA MODELS

λ_c *ARIMA Model*	*Lease Square Mean*
0,1,0	0.000002543
0,1,1	0.000002353
1,1,C	0.000001781
1,1,1	0.000001852

TABLE 13. COMPARISON OF LS MEANS FOR λ_c ARIMA MODELS

Pr /T/Ho:LS Mean (I) = LS Mean (J)

I/J	*1*	*2*	*3*	*4*
1	•	0.6641	0.0472	0.0777
2	0.6641	•	0.1209	0,1831
3	0.0472	0.1209	•	0.8281
4	0.0777	0.1831	0.8281	•

1 = 0,1,0; 2 = 0,1,1; 3 = 1,1,0; 4= 1,1,1

e. *Summary*

Since all solution failures were generated by ARIMA (1,1,0) and ARIMA (1,1,1) models, the ARIMA (0,1,0) (also known as a random walk) and the ARIMA (0,1,1) (first difference moving average) were considered to be the best models. According to Box and Jenkins[1] any ARIMA (0,1,1) can be viewed as an ARIMA (0,1,0) buried in white noise. Therefore, the ARIMA (0,1,1) was selected for use in the rest of the GAPCEEC study. In symbolic terms, this model is:

$$Y_t - Y_{t-1} = \theta_0 + a_t - \theta_1 a_{t-1},$$

where Y is cumfail/CEFH,

θ_0 is a drift constant,
θ_1 is a moving average parameter,
and the a's are random shocks to the process.

The use of the ARIMA (0,1,1) is also supported by research completed by Dr. Singpurwalla[2].

2. Forecasting Future Reliability

The first five criteria as referenced in Appendix B, part 2, are concerned with the models' ability to forecast reliability over some future time interval.

a. *Near-Term Forecast*

The . t criteria considered was the models' ability to accurately forecast the average reliability over the next 12 month time period. The following methodology was used to assess the models precision in making near-term (12 month) forecast:

[1] Box, G.E.P , and G. M. Jenkins, Time Series Analysis:Forecasting and Control, Holden-Day, San Francisco, 1976.
[2] Singpurwalla, N. D., "Time Series Analysis and Forecasting of Failure Rate Process in Reliability and Fault Tree Analysis: Theoretical and Applied Aspects of System Reliability and Safety Assessment," SIAM, 1975, pp. 483-507.

Start with the first 18 months of data and

(1) Estimate the model parameters,

(2) Calculate the predicted reliability for each of the next 12 months,

(3) Compute the model precision (RMSE, Largest Absolute Error, Mean Delta Percent Error, and Mean Absolute Error) as in subhead c.

(4) Increase the data base by 6 months (18 + 6) and repeat steps 1 through 4 above.

Each of the five models were evaluated against each of the data sets already shown in Table 7. The average precision was computed for each model as the data base is as shown graphically in Figure 10. The mean and ±2 standard deviations show the expected variation from data set to data set of the precision for each model graphically shown in Figures 11 through 15. Based on Figure 10, the AMSAA/Duane model, the EBI model and the ARIMA model all predict average reliability within ±20%. To substantiate this result statistically, each of the five models' precision (average mean % Error) was compared to identify differences. Table 14 shows the calculated values of the Z statistic (standard normal random variable) to be compared to the Normal Table (Z_u). At the 90% significance level, the tabled value (Z_u)is 2.8. Therefore, values in Table 14 greater than 2.8 indicate the models are different.

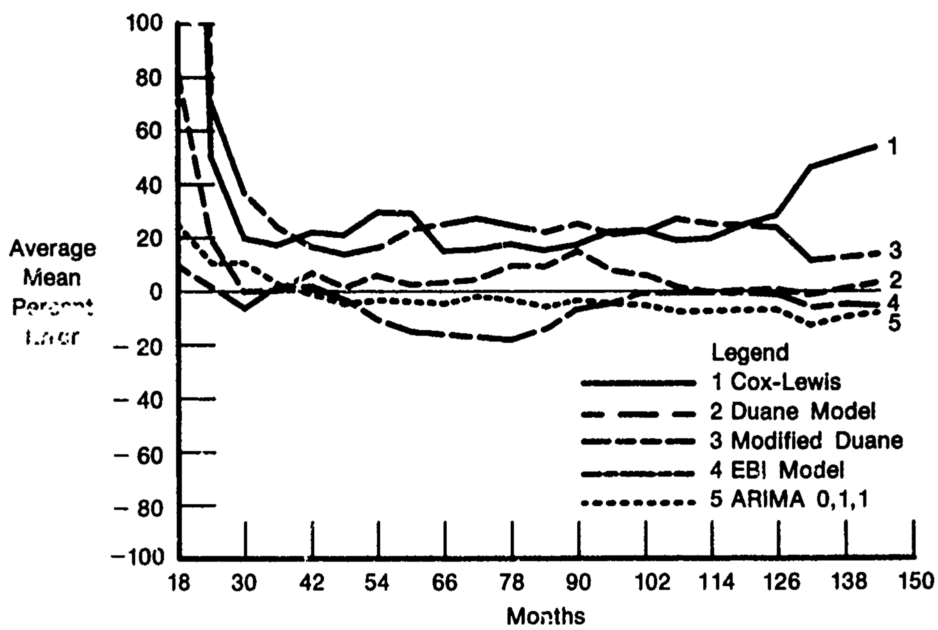

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Figure 10. Forecast Performance Over All Data Sets — All Models

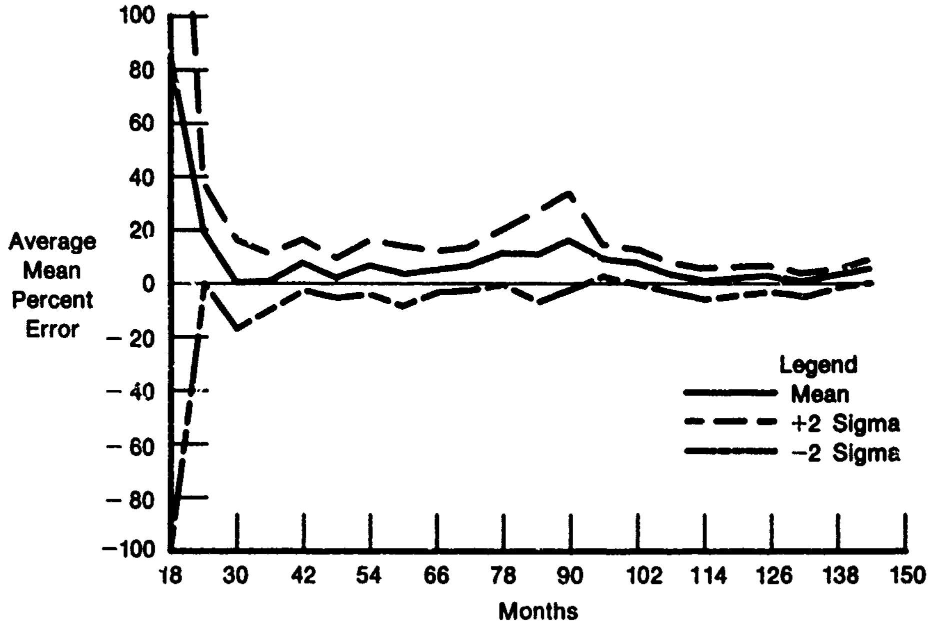

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Figure 11. Forecast Performance Over All Data Sets — Duane Model

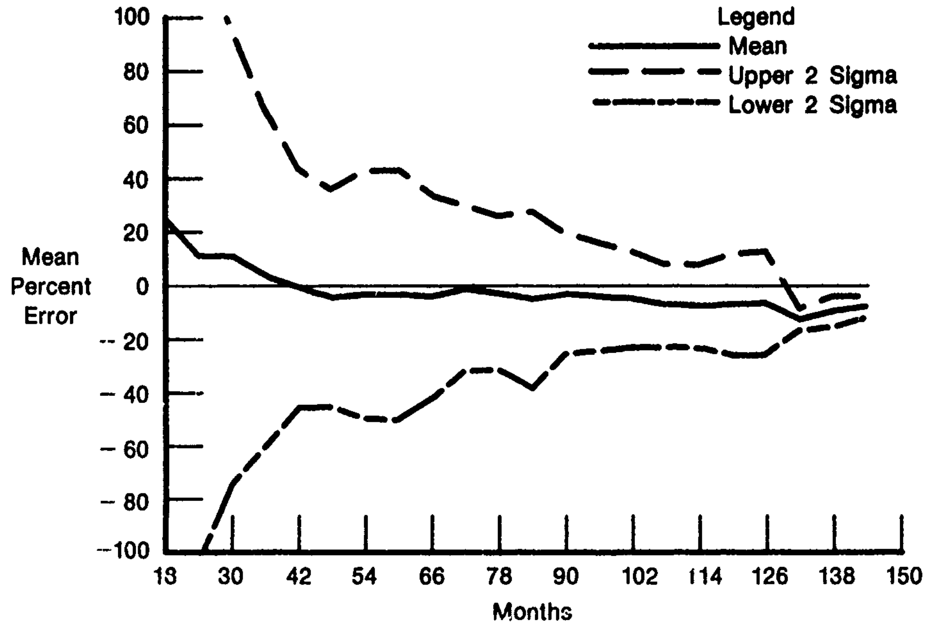

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Figure 12. Forecast Performance Over All Data Sets — Time Series

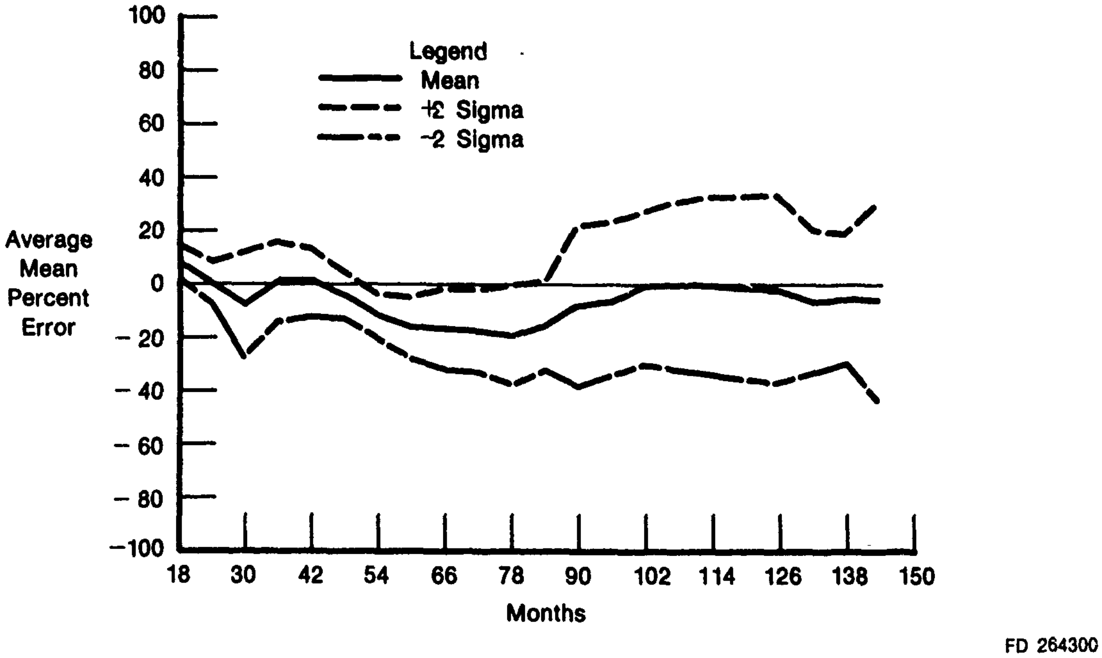

Figure 13. Forecast Performance Over All Data Sets — Endless-Burn-In Model

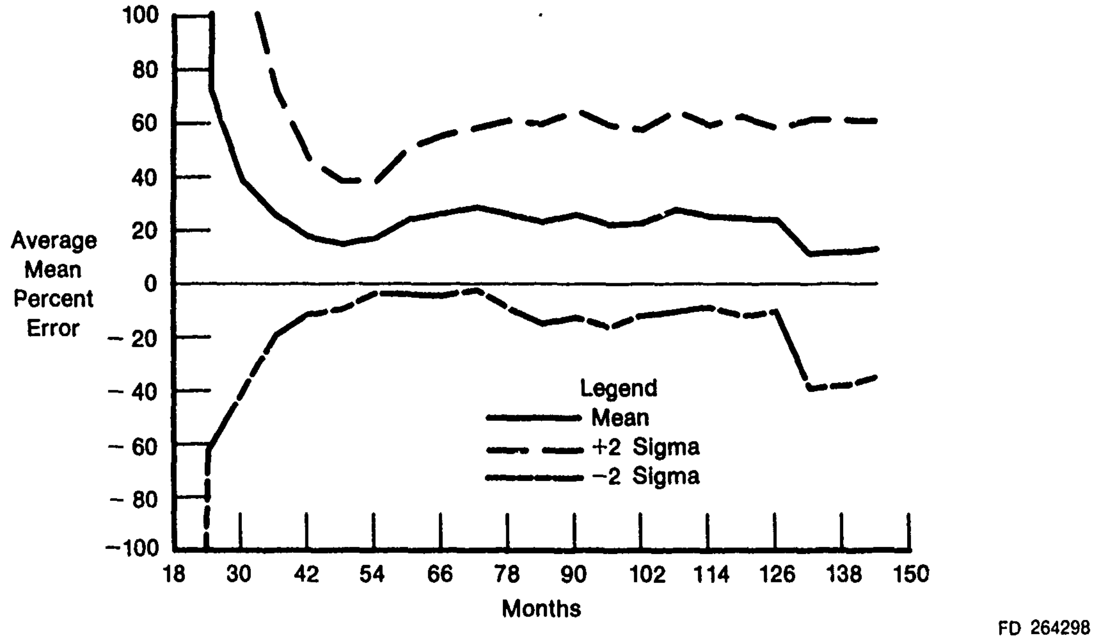

Figure 14. Forecast Performance Over All Data Sets — Modified Duane Model

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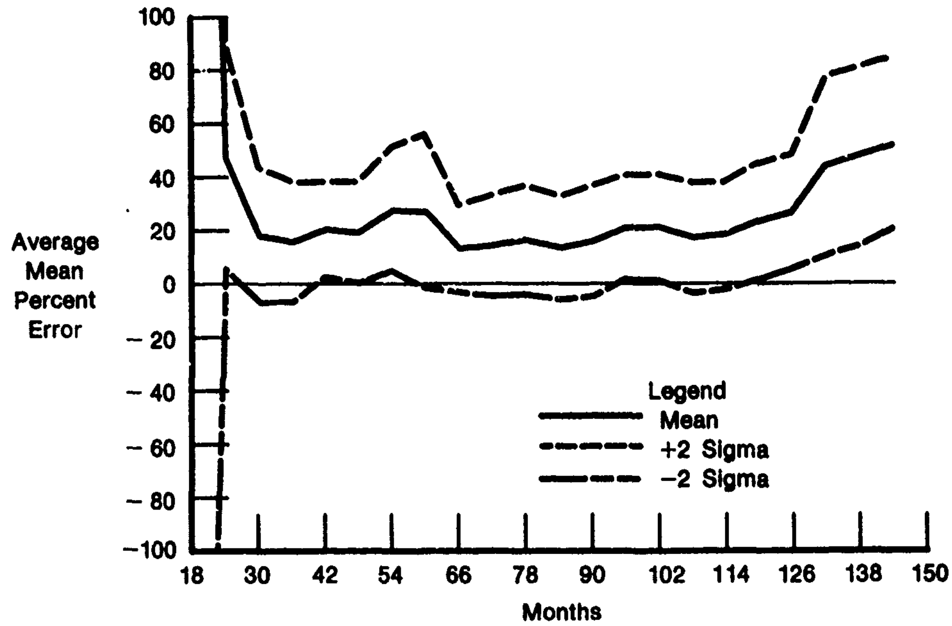

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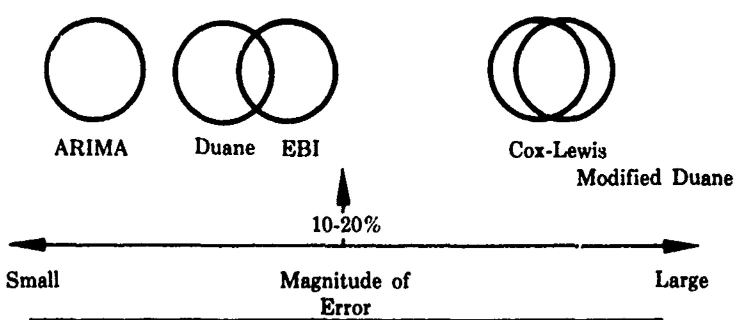
Figure 15. Forecast Performance Over All Data Sets — Cox-Lewis Model

TABLE 14. NEAR-TERM FORECASTING COMPARISON

	AMSAA/ Duane	*EBI*	*Modified Duane*	*Cox-Lewis*
ARIMA	4.3*	6.4*	13.7*	12.1*
AMSAA/Duane		2.7	7.1*	8.6*
EBI			4.3*	3.1*
Modified Duane				1.4

* Indicates that models differ at the 95% significance level.

These results can be illustrated with a Venn diagram as follows:



b. Comparison of Methods of Parameter Estimation

The Modified Duane model was *expected* to forecast with as good or better precision than the AMSAA/Duane model. This was expected because the Modified Duane model would be mathematically equivalent to the AMSAA/Duane model if the limiting parameters were zero.

- Modified Duane model

$$\rho(t) = \lambda\beta T^{\beta-1} + \theta$$

- AMSAA

$$\rho(t) = \lambda\beta T^{\beta-1}$$

Since the Modified Duane model *did not* forecast with the same precision (or better) than the AMSAA/Duane model, and since the only obvious difference was in the method of parameter estimation, a comparison was made of the AMSAA models forecasting precision using (1) regression estimators and (2) maximum likelihood (ML) estimators. The ML estimators are the estimates which are *most likely* (from a probability point of view) to have generated the observed data. ML estimators usually fit the most recent data better than regression estimates. The ML estimators were found to be much better for forecasting future reliability (Figures 16 and 17). The ML estimates for the AMSAA/Duane model, when working with grouped data, are presented in MIL-HDBK-189, Appendix D. It would be necessary to solve a system of nonlinear equations to find ML estimates for the modified Duane and Cox-Lewis models (Appendix C). Because of the difficulty in obtaining ML estimates for the Cox-Lewis and Modified Duane models, regression estimates were used. This is one possible reason for their poor performance.

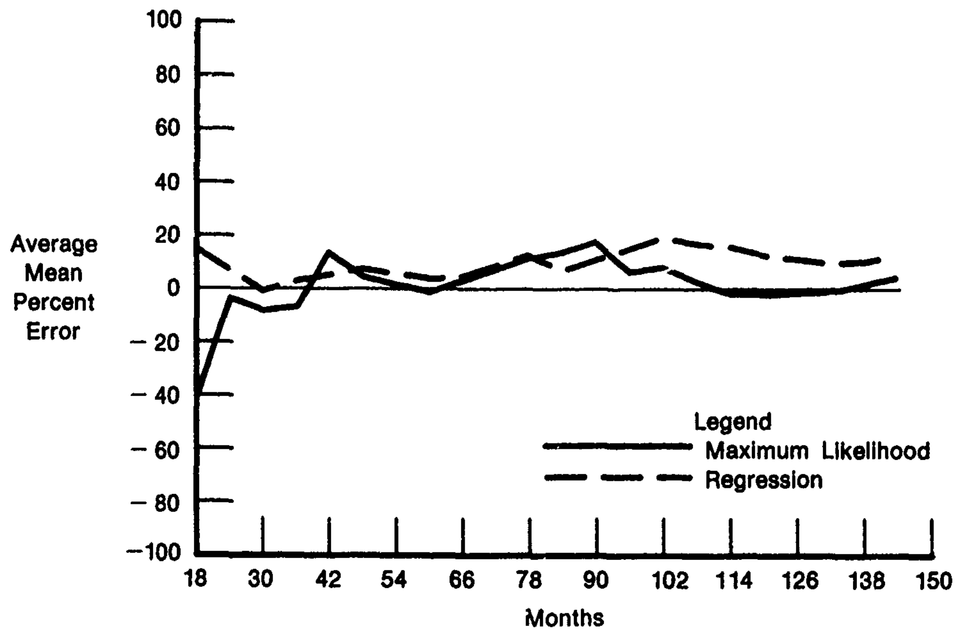


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Figure 16. Comparison of Methods of Parameter Estimation

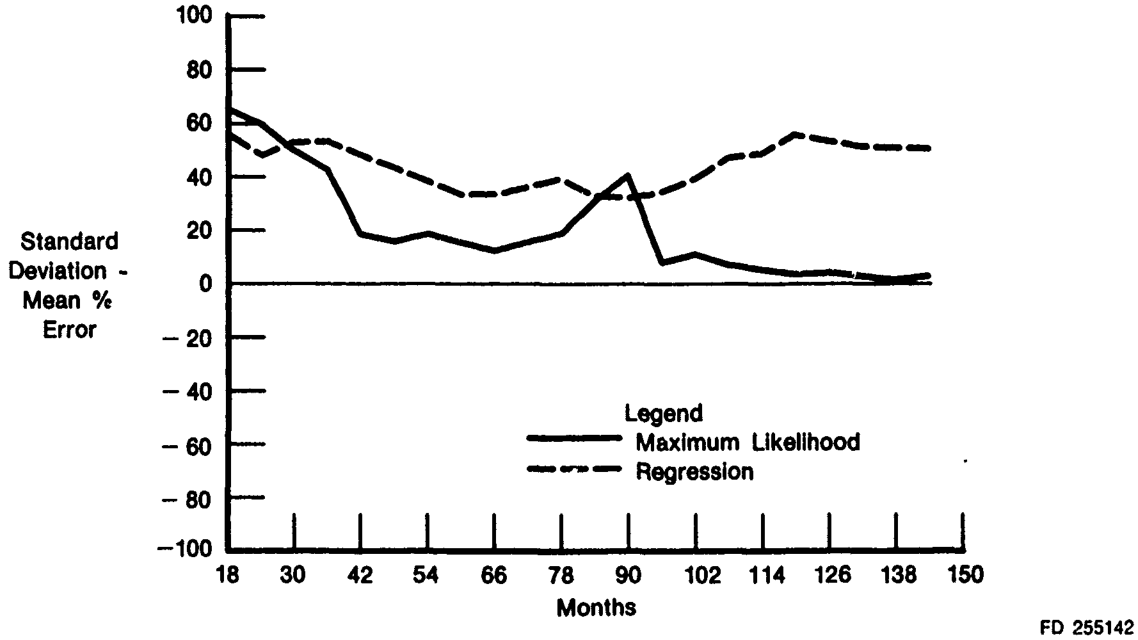

Figure 17. Comparison of Methods of Parameter Estimation

c. Long-Term Forecast

Criteria 2 through 4 (Appendix B, Part 2) are concerned with the models ability to forecast the reliability of a specific time point (entry into the field) and over a long period of time (from development to maturity). To address these questions, a study was undertaken with the following objectives:

(1) To determine the precision with which each model could forecast reliability at entry into the field and project reliability at maturity from *development data.*

(2) To determine the lead time required to project reliability at entry into the field and project reliability at maturity within $\pm 10\%$.

The methodology used for the first objective (predicting field entry and maturity from development) was as follows:

(1) Break the data into two parts, *total experience prior to entry into the field,* and *field experience* shown in Figure 18.

(2) Estimate the model parameters using the prior to entry into the field experience.

(3) Forecast points for each 6th-month of field experience.

(4) Calculate the percent error for each point

$$\left[\frac{\text{Forecast} - \text{Actual}}{\text{Actual}}\right] *100$$

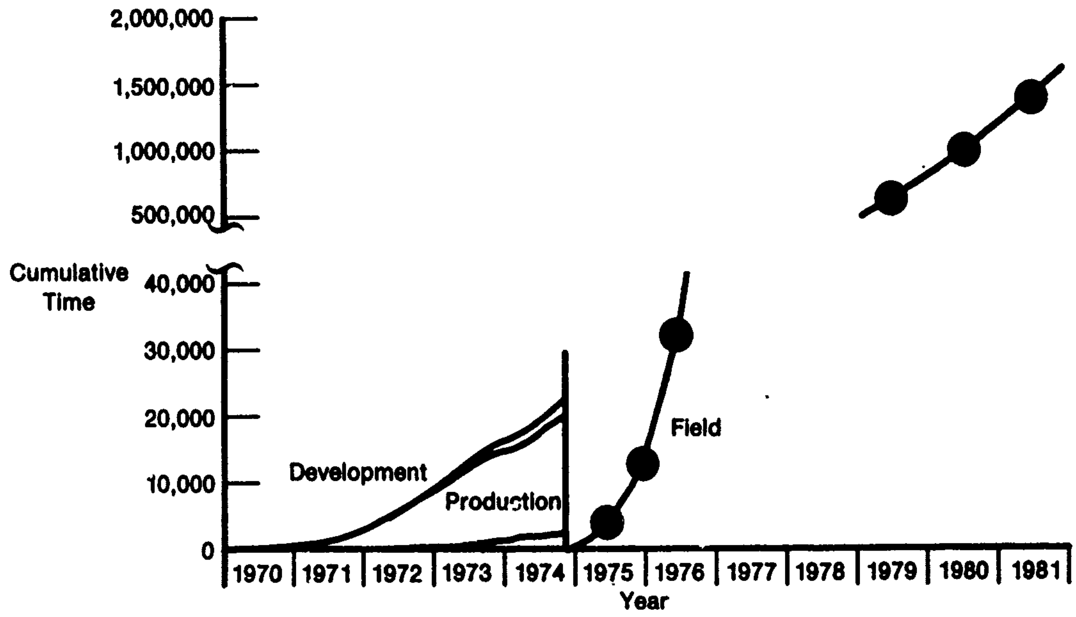

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Figure 18. Long-Term Forecast

Entry into the field (30,000 hr) was the first prediction made (6 month) and maturity (1 million hr) was month 68. Figures 19 through 23 show the results of this study for each model with each component. The AMSAA/Duane model and Time Series did the best (within ±20%) at entry into the field. This is consistent with near-term forecasting results. None of the models did an acceptable job of forecasting to maturity from development.

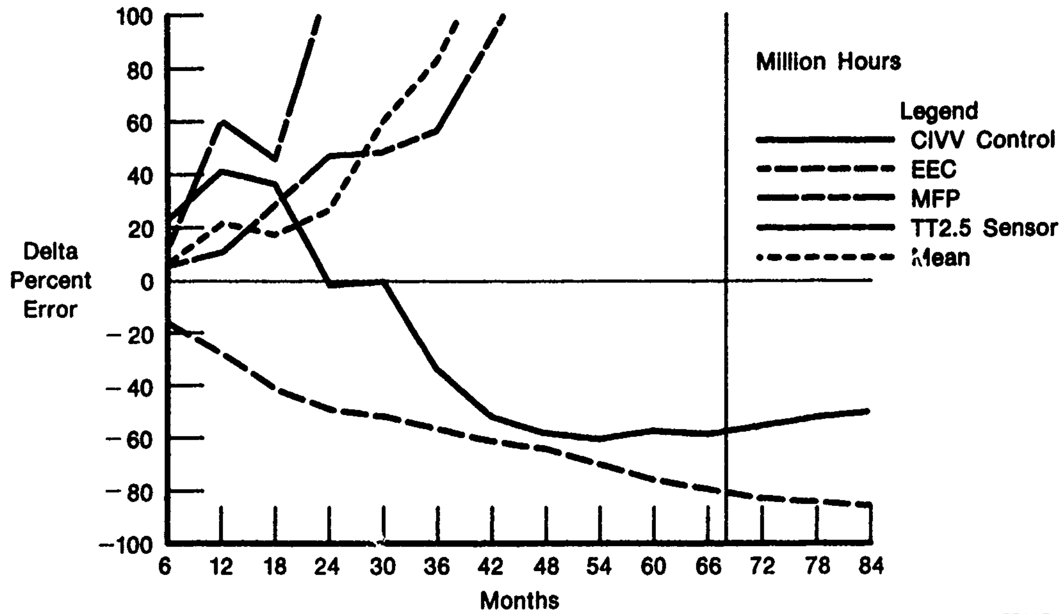

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Figure 19. Duane Analysis Long-Term Forecast

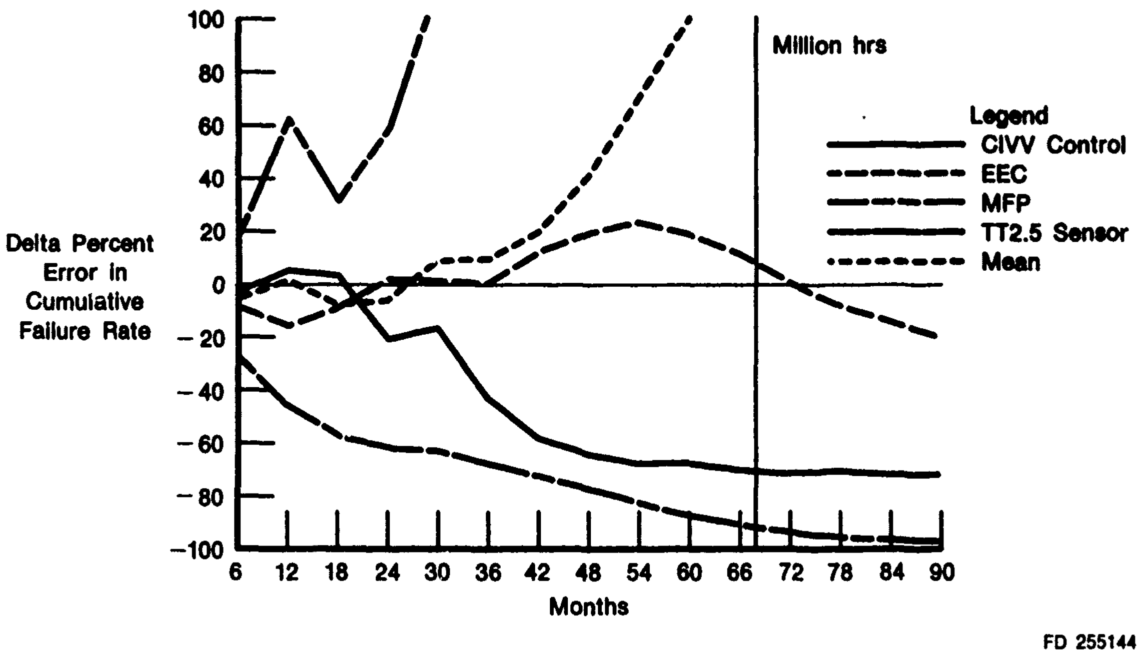

Figure 20. Time Series Analysis Long-Term Forecast

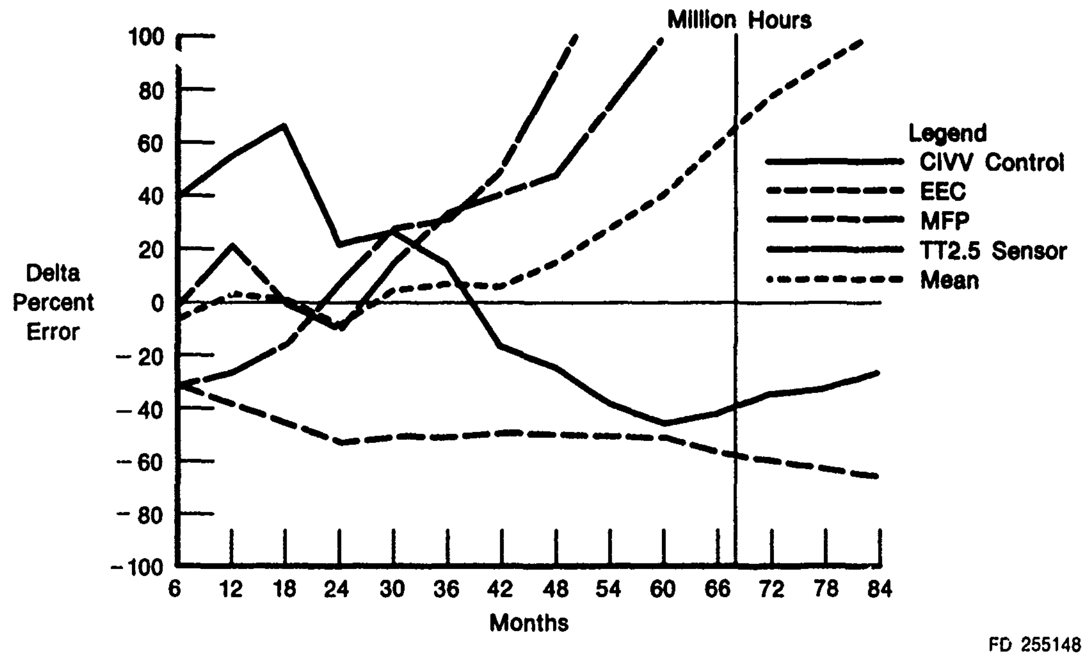

Figure 21. Endless-Burn-In Analysis Long-Term Forecast

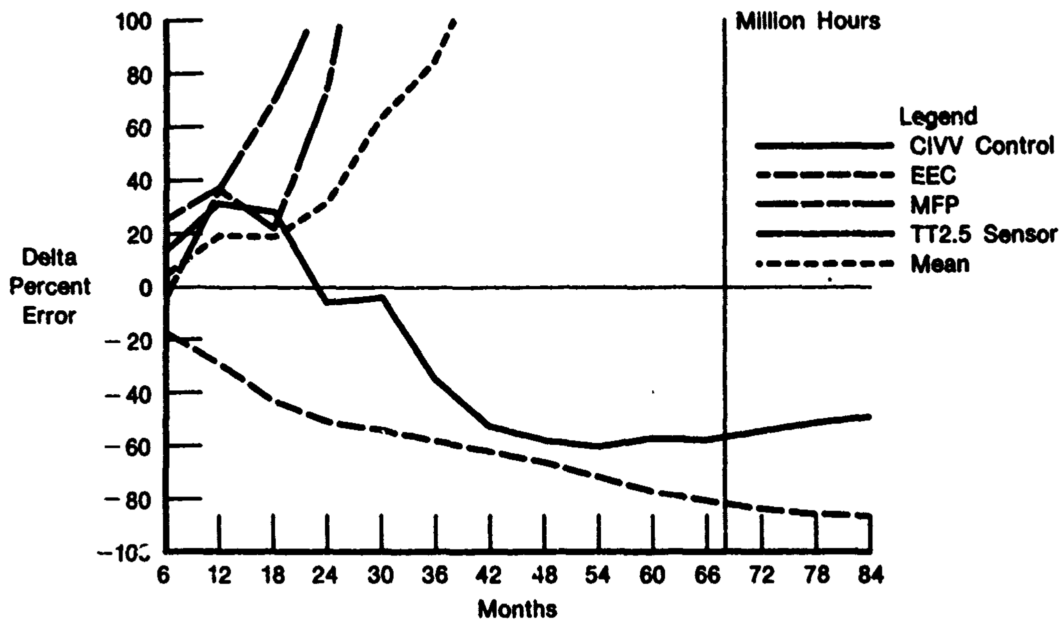

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Figure 22. Modified Duane Analysis Long-Term Forecast

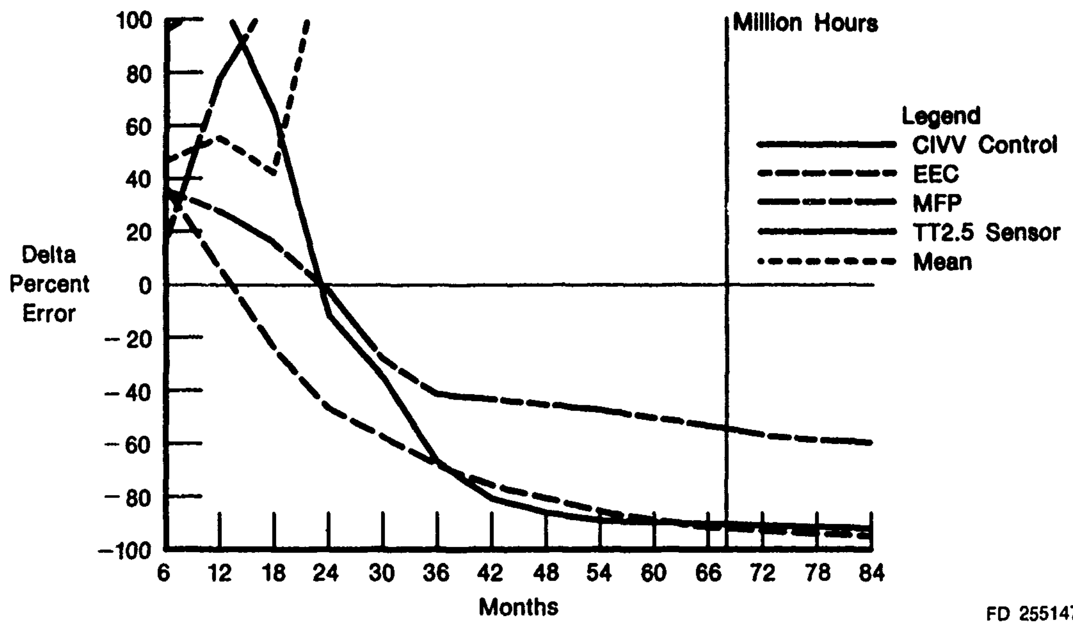

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Figure 23. Cox-Lewis Analysis Long-Term Forecast

The methodology used for the second objective, determining required lead time, was as follows:

Starting with the first 18 points,

(1) Estimate the model parameters from the data.

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(2) Forecast the reliability at entry into the field (30,000 hr) and maturity (1 million hr).

(3) Compute the percent error as before.

(4) Increase the sample size by one point (move closer to the entry point or maturity) and repeat steps 1 through 4 above.

Figures 24 through 28 show the results of lead time required to forecast entry into the field (30,000 hr). The ARIMA model and the AMSAA/Duane model were able to forecast within ±10% 2 to 3 months in advance. The other models were not successful at obtaining these goals. Figures 29 through 33 show the results of lead time requirements for forecasting maturity. The ARIMA and Duane models were able to forecast within ±10% approximately 1 year prior to maturity.

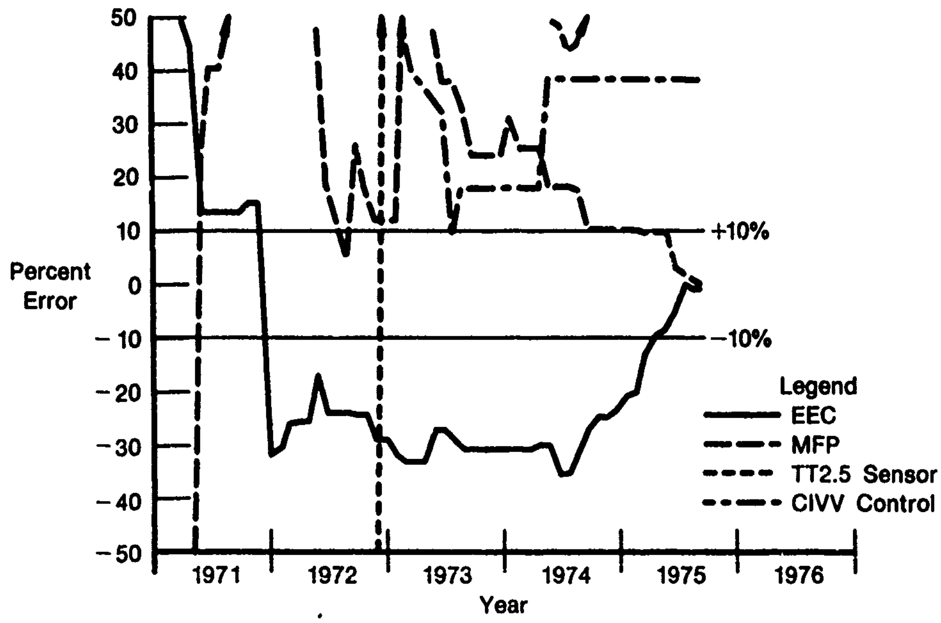

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Figure 24. Duane Model Forecast at 30,000 Hours

Long-term forecasting with the EBI model was attempted only with the EEC data set since it was the only complete data set that had solid state electronics failures. These failures were sorted out and modeled separately.

In general, all of the models were unreliable in forecasting reliability beyond 1 to 2 years into the future. As a result of all of the models poor performance in long-term forecasting, a method of long-term forecasting based on management goals, engineering and statistics is recommended (Appendix D).

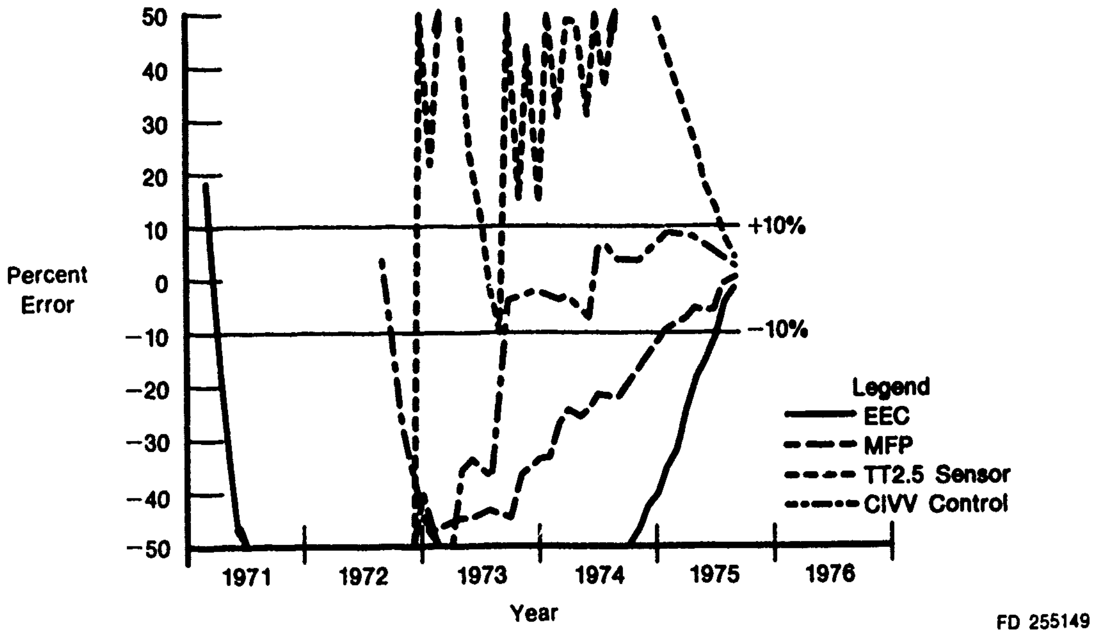

Figure 25. Time-Series Model Forecast at 30,000 Hours

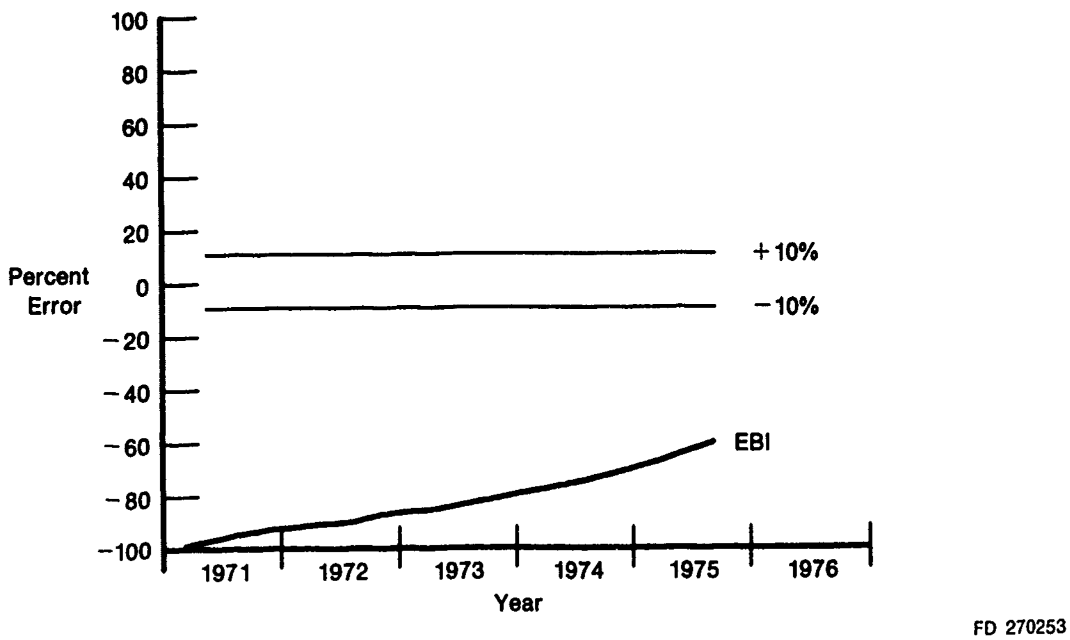

Figure 26. Endless-Burn-In Model Forecast at 30,000 Hours

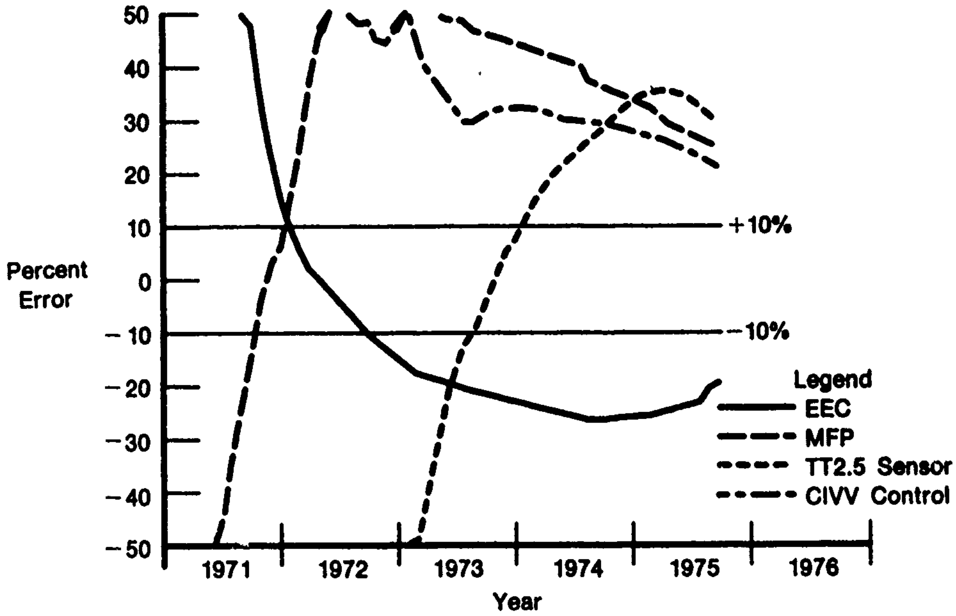

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Figure 27. Modified Duane Model Forecast at 30,000 Hours

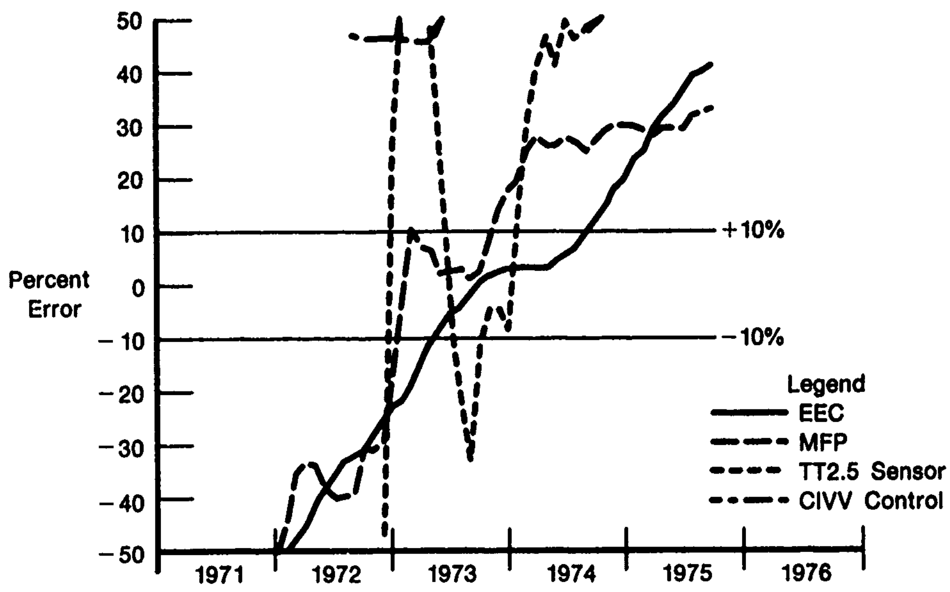

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Figure 28. Cox-Lewis Model Forecast at 30,000 Hours

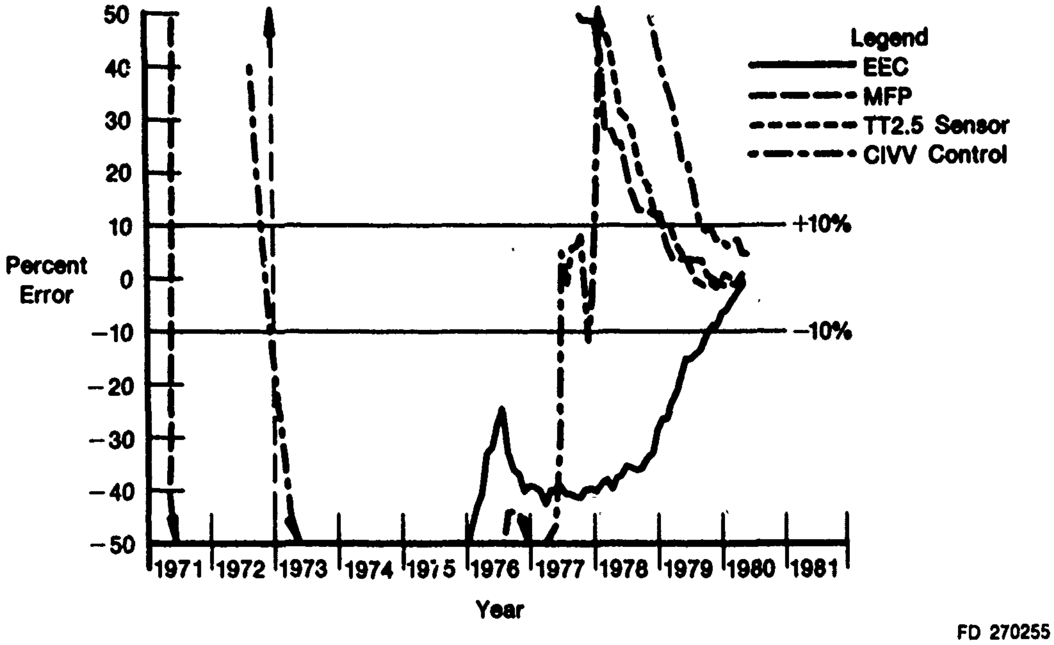


Figure 29. Duane Model Forecast at 1 Million Hours

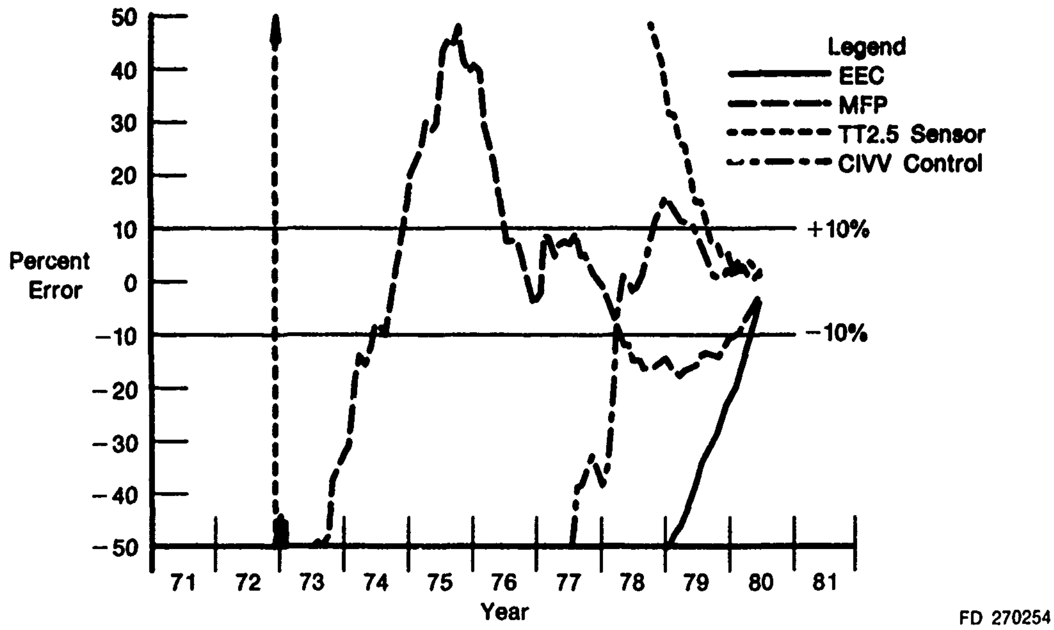


Figure 30. Time-Series Model Forecast at 1 Million Hours

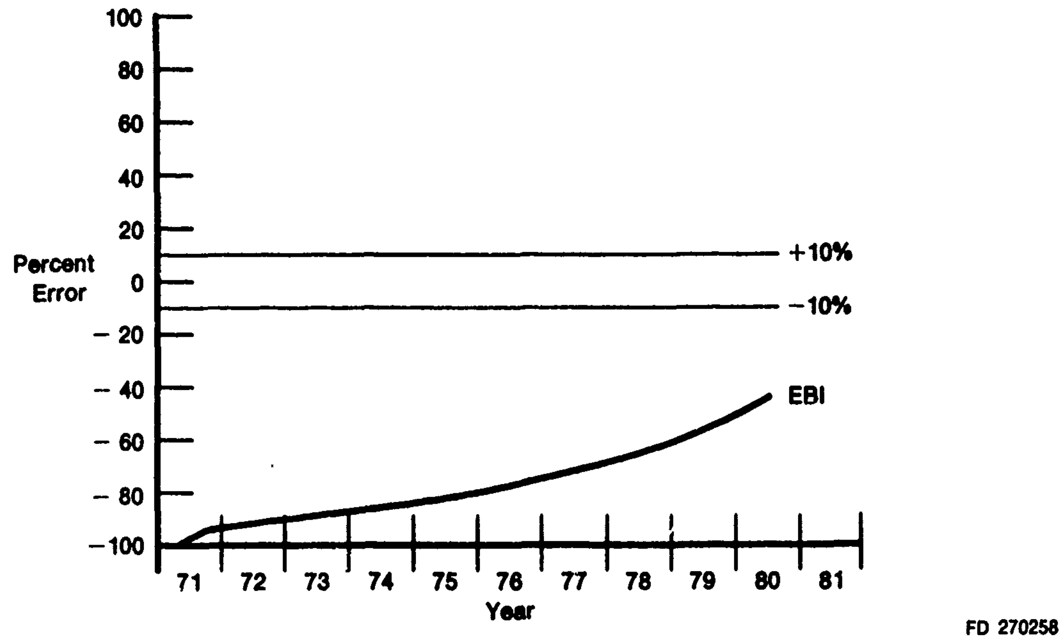


Figure 31. Endless-Burn-In Model Forecast at 1 Million Hours (EEC Operating Data)

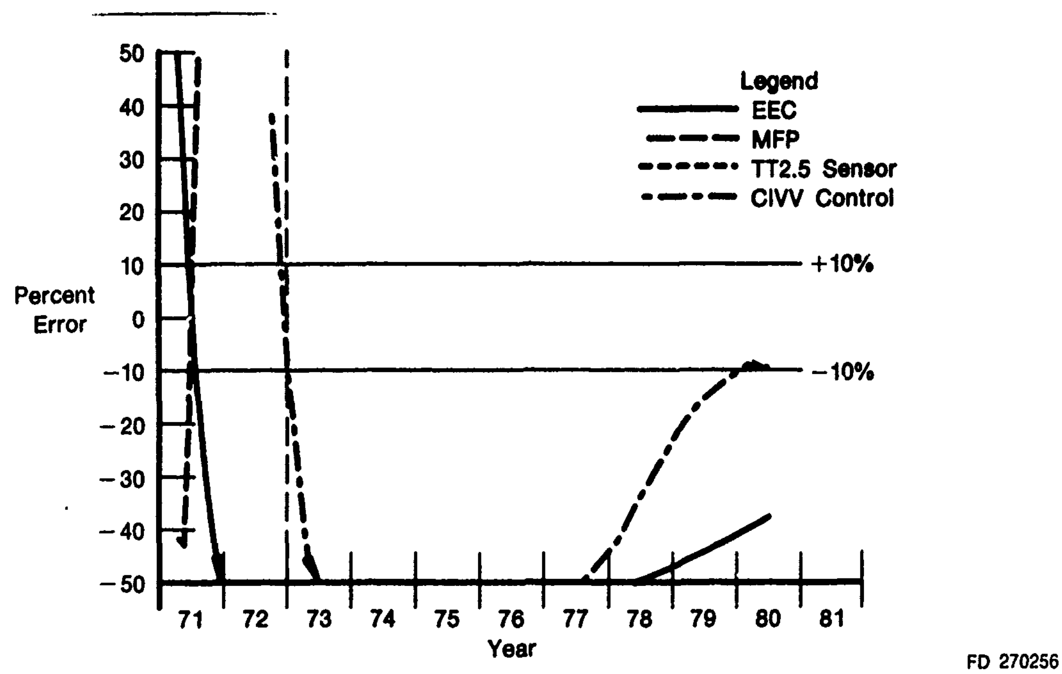


Figure 32. Modified Duane Model Forecast at 1 Million Hours

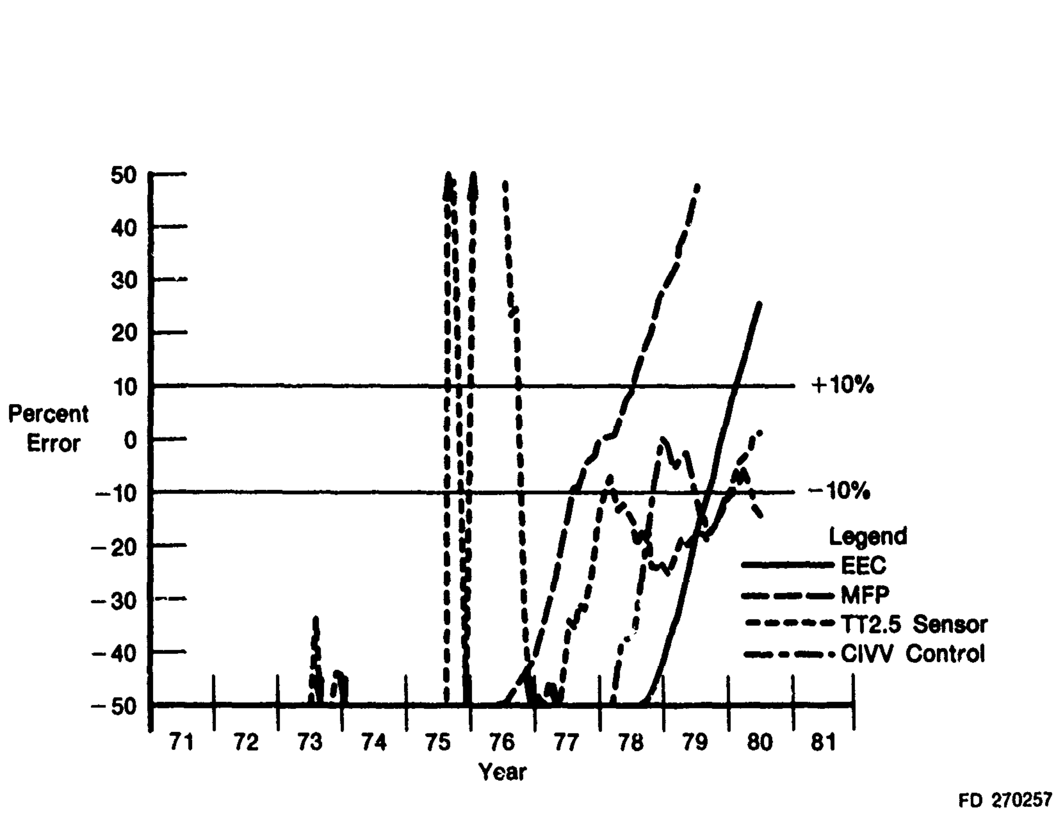


Figure 33. Cox-Lewis Model Forecast at 1 Million Hours

3. Overall Model Comparison

a. Development of Criteria

Since the value of each model relative to specific criteria (Appendix B, Part 1) is somewhat subjective, it was necessary to rewrite the criteria to make them consistent with a six-point rating scale frequently used in the behavioral sciences[1] to evaluate subjective data such as interest, values, attitudes, opinions, etc.

Each criterion statement must be rewritten so that the statement itself is relatively neutral and can be answered completely by one of the six responses to make each statement consistent with the rating scale (Appendix B, Part 2). Two of the original criteria had to be split into two statements each (criteria 4 and 6) and two were combined into a single statement (criteria 10 and 11). Criterion 4, Physical Interpretation, could be answered one way for interpretation of parameters and another way for interpretation of the model. A similar problem was encountered with criterion 6, Adaptability. Criteria 10 and 11 were combined into one statement because the results of criterion 10 strongly influences the results of criterion 11. This is because most test for trends (criterion 11) involve influences (confidence limits) on the growth parameter.

b. Analysis and Methodology Used to Compare the Models

Each statement was assigned a value for each model based on experience and technical information obtained in the literature search (Appendix A). Table 15 shows the response to each criterion for each model. These data in Table 15 were then transformed into Van der Waerden scores[2] and a multiple comparison analysis was done based on Friedman's rank sums[3]. The Van der Waerden scores were used because of the small sample size (less than 30 statements). Friedman's rank sums is a distribution free method of comparing all pairs of models simultaneously.

---

(1) Nunnally, J. C., Psychometric Theory, McGraw-Hill, Inc., 1967, p. 520.
(2) "SAS Institute Technical Report S-120X," SAS Institute, Inc., Cary N. C., 1981.
(3) Hollander, M., and D. A. Wolfe, Nonparametric Statistical Methods, John Wiley & Sons, Inc., New York, p. 138.

TABLE 15. RESPONSE* OF MODELS TO CRITERIA

Table 1 Statement	*ARIMA*	*Duane*	*Endless Burn-In*	*Modified Duane*	*Cox-Lewis*
1	5	5	5	2	2
2	2	2	2	1	1
3	2	2	1	1	1
4	2	3	1	1	1
5	2	5	2	2	2
6	4	5	5	4	4
7	2	5	4	4	4
8	1	5	5	5	4
9	1	5	5	4	4
10	5	3	3	3	2
11	6	4	2	4	4
12	5	5	2	2	2
13	5	2	2	2	2
14	1	1	5	5	2
15	4	5	4	2	3

* Possible responses to criteria:

1. Completely disagree
2. Mostly disagree
3. Slightly disagree
4. Slightly agree
5. Mostly agree
6. Completely agree.

c. *Results*

The AMSAA/Duane model had the highest over all score, but it was not significantly different statistically from the ARIMA model. The EBI model was similar to the ARIMA model but not as effective as the AMSAA/Duane model. The EBI model scored lower than the AMSAA/Duane model on statements involving sample size requirements (criterion 5) and physical restrictions (criterion 11). The Modified Duane model and Cox-Lewis model did not perform as well in the forecasting statements (1 through 5). This is due in part to the lack of estimation techniques that are available for those models when the data are grouped by month. Table 16 shows the results of the simultaneous pairwise comparison. Values inside the interval (−1.65, 1.65) indicate the models are not different.

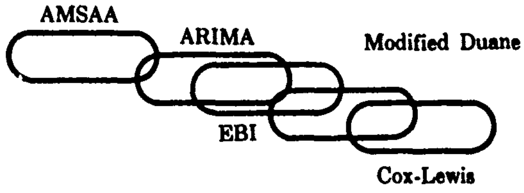
C. RECOMMENDATIONS FROM EVALUATION

As a result of the work done in the model evaluation phase of this contract, the AMSAA model was considered to be the most appropriate model for future consideration with the ARIMA model as an alternative (and in some cases the EBI). Refer to Appendixes D and E for more information relative to selection and implementation of these models.

TABLE 16. RESULTS OF MODEL COMPARISON

	AMSAA/ Duane	*EBI*	*Modified Duane*	*Cox-Lewis*
ARIMA	−1.55	0.202	1.46	2.02
AMSAA/Duane		1.75	3.02	3.58
EBI			1.26	1.82
Modified Duane				0.56

These results are illustrated with a Venn diagram



If two models overlap, they are not different at the 90% significance level.

SECTION IV

PROGRAM SYNTHESIS

A. INTRODUCTION

The program synthesis phase of this study consisted of two primary objectives: (1) To research confidence limits and goodness-of-fit tests available for the AMSAA and Time Series models; and (2) To develop a procedure to be used as a guide for applying the AMSAA and Time Series model to a development program.

B. CONFIDENCE LIMITS AND GOODNESS-OF-FIT TEST

A thorough review of the literature published within the past 10 years on confidence limits and goodness-of-fit tests for the Weibull process (AMSAA model) and Box-Jenkins Time Series modeling was completed (see Reference Subsection 4). The confidence limits and goodness-of-fit tests recommended in the GAPCEEC study are consistently used and referenced in the literature. The following are recommended:

1. For the AMSAA Model, With Time-Truncated Data

The exact unit and fleet time is known at each failure and some test time has been run since the last failure. The following are recommended:

- Goodness-of-fit test. The Cramer von-Mises test is recommended (Appendix D). Reference 7, pp. 70 and 135.

- Confidence limits for the model parameters, β and λ. Reference 7, p. 68 and Reference 5, respectively.

- Test for growth. Use the confidence limits on the β as illustrated in Reference 7, p. 68.

- Confidence limits on reliability. Confidence limits are available for the instantaneous MTBF. Reference 2.

2. For the AMSAA Model, With Grouped Data

Failures and fleet time tracked in discrete intervals, for example, by month. The following are recommended:

- For goodness-of-fit and test for growth, use a Chi-Square test. Reference 7, pp. 68 and 140, and p. 69, respectively.

- There are no easy to apply techniques for confidence limits on the parameters or on reliability available at this time.

3. For Box-Jenkins Time Series Analysis

There is no goodness-of-fit test for a specified model (first difference moving average model). There are tests available to determine if an unspecified model is needed. The following test and confidence limits are recommended:

- Model identification. A Chi-Square Test is available to test the hypothesis that the series is white noise (no model needed). Once a model is selected, for example, moving average model, this test can be used to determine if the residuals are white noise (model explains all meaningful variation). Reference 13, p. 297.

- Confidence limits are available for the drift parameter, the moving average parameter and the cumulative failure rate. Reference 8, pp. 194-195, 187-188, and 144-146, respectively.

C. PROCEDURE FOR USING THE AMSAA MODEL

In the procedure (Appendix D) there are five sections and an Appendix (Appendix E). The first three sections deal with planning and management of the growth modeling process, while the last two sections are concerned with the analysis once the program is underway and the data has been collected.

1. Section I — Preliminary Analysis and Background Research

Section I gives a description of the background work necessary to plan a reliability growth program. This section covers breaking the data into test phases, reviewing past growth models and growth rates, assessing the computer capabilities and a description and recommendation of what kind of data to collect.

2. Section II — Model Selection

Section II gives a detailed description of each of the following models: The AMSAA/Duane model, the Box-Jenkins Time Series model and the Endless-Burn-In model. For each of these models, the advantages of the model are presented as well as the form of the model and the assumptions of each model. The AMSAA/Duane model is generally the preferred model.

3. Section III — Design the Ideal Growth Curve

Section III is concerned with illustrating how to draw the planned curve from the information obtained in the first section of the procedure. This section assumes that the AMSAA/Duane model is being used. After illustrating how to calculate the growth parameter and average MTBF for each phase, Section 3 discusses how to evaluate the feasibility of the *ideal curve*.

4. Section IV — Analysis of Data

Section IV illustrates how to perform engineering and statistical analysis to assess the current reliability and to forecast reliability at points in the future. This section assumes that the data has already been gathered. The engineering analysis subsection discusses reviewing the failure data from an engineering perspective to determine the type of failure mode, and the cause of the failure. Engineering assessment of the influence of potential fixes for each failure mode is discussed. Statistical analysis of both grouped data the time-truncated data are reviewed. This analysis includes: preliminary analysis, parameter estimation, confidence limits, goodness-of-fit

test and tests for growth, as well as estimation of current and future reliability. Development and operational environments are considered in this section.

5. Section V — Interpretation of Results

This section briefly describes the interpretation of the results of the statistical analysis in light of the original objectives: assessment, prediction and control.

D. REFERENCES, BOX-JENKINS TIME SERIES

This list of references to the procedure is intended to introduce some of the basic concepts of ARIMA modeling to the user and to illustrate its applicability to reliability-growth modeling. There are two main sections, the first being a discussion of the background, motivation for and theory of time series, and the second being an example of an ARIMA model applied to part of the EEC data.

Articles Relating to Confidence Limit of the AMSAA and ARIMA Models

1. Belot, E. F., "A Computer Program for Estimation of Parameters of the Weibull Intensity Function and for the Cramer von-Mises Goodness-of-Fit Test," *Technical Report No. 279*, Aberdeen Proving Ground, Md.

2. Crow, L. H., "Confidence Interval Procedures for the Weibull Process with Applications to Reliability Growth," *Technometrics*, Vol. 24, 1982.

3. Crow, L. H., "Confidence Interval Procedures for Reliability Growth Analysis," *Technical Report No. 197*, Aberdeen Proving Ground, Md., June 1977.

4. Crow, L. H., "Reliability Analysis of Complex Repairable Systems," *Soc. Industrial and Applied Mathematics, Reliability and Biometry: Proceedings of Statistical Analysis of Life Length*, July 1974.

5. Finkelstein, J. M., "Confidence Bounds on the Parameters of the Weibull Process," *Technometrics*, Vol. 18, No. 1, 1976.

6. Lee, L., and S. K. Lee, "Some Results on Inferences for the Weibull Process," *Technometrics*, Vol. 20, 1978.

7. MIL-HDBK-189, "Reliability Growth Management," 1981.

8. Box, G. E. P., and G. M. Jenkins, *Time Series Analysis: Forecasting and Control*, Holden-Day, San Francisco, 1976, pp. 91-108, 119-120, 135, 144-146, 176, 187-189.

9. Fuller, W. A., *Introduction to Statistical Time Series* , John Wiley & Sons, New York, 1976, pp. 372-373.

10. Anderson, O. D., *Time Series Analysis and Forecasting* , Butterworths, London, 1976, pp. 93-123.

11. Cleary, J. P., and H. Levenbach, "The Professional Forecaster," *Lifetime Learning Publications*, Belmont, CA, 1982, pp. 263-282.

12 Singpurwalla, N. D., "Time Series Analysis and Forecasting of Failure Rate Process in Reliability and Fault Tree Analysis: Theoretical and Applied Aspects of System Reliability and Safety Assessment," *SIAM,* Philadelphia, 1975, pp. 483-507.

13. Ljung, G. M., and G. E. P. Box, "A Measure of Lack of Fit in Time Series Models," *Biometrika,* Vol. 65, No. 2, 1978.

SECTION V

WARRANTY CONSIDERATIONS

The contribution that growth modeling can make to warranty of future engine components was investigated as part of this program. Two types of warranties have been used for engine and engine components: 1) warranty of a level of reliability (mean time between failure (MTBF)) at a specific point in development; and 2) warranty of a specified period of operating hours with failures occuring during that period repaired at no cost to the customer. These two types of warranties, and the growth models contribution to each, are discussed further below.

A. MTBF WARRANTY

Warranty of a specific level of reliability has been applied to engine programs during the development process. This type warranty guarantees that a certain level of reliability, in terms of MTBF, will be met at the end of a particular development phase. During development, detailed data can be collected and analyzed.

The reliability-growth modeling techniques, which have been developed in this study, apply directly to tracking of this development data, assessment of current (instantaneous) MTBF and short-term projection of reliability. Using these techniques, the reliability-growth model can be used to evaluate the ability of the current design to meet the warranty value at the required point in development. The current and projected reliability using the model do not provide the statistical confidence provided by a demonstration type test and do not replace the requirement to demonstrate contractual compliance with specific warranty values.

B. FIXED TIME WARRANTY

Warranty for a specific number of operating hours following delivery has been applied to engine programs during operational field usage. This type warranty covers malfunctions of a component/engine for a specified number of operating hours from the date of delivery to the customer. Provisions of this type warranty could include repair at no cost, replacement, reimbursement for repair or others as agreed to. Knowing the past history of a component, the current and projected reliability of the component can be calculated using the reliability-growth model. The current or projected reliability in turn can be used to calculate the expected returns under a given warranty.

For this type warranty, an evaluation procedure to use the growth model with existing component data was established. The purpose of this evaluation was to determine the accuracy of model projections of *warranty* returns versus actual field experience.

Warranty Evaluation Procedure

The procedure for evaluating the growth model's ability to project returns under a fixed time type warranty situation consisted of:

(1) Assuming a warranty situation

(2) Projecting component failures based upon prior data, using the reliability-growth model

(3) Determining the actual number of failures from the failure data base

(4) Comparing the projected number of failures to the actual number of failures during the assumed warranty.

One warranty selected was for the first 200 hours of control component field operation. This is similar to the warranty currently in place for F100 engine components. In addition, warranty periods of the first 300 and 400 hours were used in the evaluation.

Each assumed warranty period was applied to the yearly production lots of components from development through current production. Knowing the warranty hours per unit and the number of units in each production lot allows calculation of the total hours of exposure for the lot. For example, if there were 100 units in a given production lot and a 200 hour warranty, a total of 20,000 hours of operation are covered by the warranty.

Component data up to the delivery of the first warranty units was used in making the MTBF projection. The data was fit to the reliability-growth model, model parameters determined, and a calculation of instantaneous MTBF made at delivery of the first warranty units. This instantaneous MTBF was used to calculate the expected number of returns.

Knowing the total hours covered by warranty and the instantaneous MTBF, the expected returns are calculated as hours divided by MTBF. These calculations were made for each year's production lot considering all available experience up to that point in time. For example, the first year's warranty used only development experience, the second year's warranty used development, data and the first year's production experience, and the third year's warranty used development plus two years production, etc.

The component data base was used to determine the actual number of failures from each production lot under the warranty ground rules. These actual failures were then compared to the projected failures to determine the accuracy and hence the usefulness of the model with regard to warranty evaluation.

C. WARRANTY EVALUATION RESULTS

Tables 17, 18, and 19 show the results of this evaluation. In these tables, PRODUCTION LOT identified as *development* includes those units used for flight test evaluation; *1st Year* includes those units in the first year's production and etc. NO. UNITS is the number of units in the production lot. TOT HR is the total hours covered by the warranty calculated as No. units times warranty period. INST MTBF is the current MTBF from the growth model using all available data. Projected failures are the number of expected failures for the lot under the given warranty calculated as TOT HR divided by INST MTBF. The OBSERVED FAILURES are those that actually occurred with hours less than the warranty limit.

TABLE 17. SIMULATED 200 HOUR WARRANTY RESULTS

Model Projection					*Field Data*
Production Lot	*No. Units*	*Total Hr*	*Inst MTBF*	*Projected Failures*	*Observed Failures*
Development	76	15,200	1,380	11	33
1st Year	173	34,600	608	57	59
2nd Year	362	72,400	576	126	74
3rd Year	426	85,200	813	105	72
4th Year	293	58,600	1,014	58	43
5th Year	511	102,200	1,268	81	58
6th Year	582	116,400	790	147	44
7th Year	796	159,200	678	235	Incomplete Data
8th Year	668	133,600	816	164	Incomplete Data

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TABLE 18. SIMULATED 300 HOUR WARRANTY RESULTS

Model Projection					*Field Data*
Production Lot	*No. Units*	*Total Hr*	*Inst MTBF*	*Projected Failures*	*Observed Failures*
Development	76	22,800	1,380	17	39
1st Year	173	51,900	608	85	76
2nd Year	362	108,600	576	189	110
3rd Year	426	127,800	813	157	120
4th Year	293	87,900	1,014	87	58
5th Year	511	153,300	1,268	121	81
6th Year	582	174,600	790	221	65
7th Year	796	238,800	678	352	Incomplete Data
8th Year	668	200,400	816	246	Incomplete Data

TABLE 19. SIMULATED 400 HOUR WARRANTY RESULTS

Model Projection					*Field Data*
Production Lot	*No. Units*	*Total Hr*	*Inst MTBF*	*Projected Failures*	*Observed Failures*
Development	76	30,400	1,380	22	46
1st Year	173	69,200	608	114	98
2nd Year	362	144,800	576	251	138
3rd Year	426	170,400	813	210	171
4th Year	293	117,200	1,014	116	91
5th Year	511	204,400	1,268	161	103
6th Year	582	232,800	790	295	78
7th Year	796	318,400	678	470	Incomplete Data
8th Year	668	267,200	816	327	Incomplete Data

From these results it can be seen that the model does not accurately predict warranty failures. This is largely due to trends of the growth curve which cause the projected MTBF to be significantly different from the actual MTBF. Figure 34 shows a plot of the 2nd years production lot projection as an example. The projection is based upon all experience through the 1st year. The value of current MTBF from the model is found to be 576 hours. This value is used for warranty return calculations. The actual MTBF as shown on the plot increased rapidly during the 2nd year resulting in a much higher MTBF than originally predicted. This results in significantly fewer returns than were projected.

The reliability-growth model can be used as a tool in predicting fixed time type warranty returns; however, the accuracy of these projections is not high and caution should be exercised.

It should also be noted that the model projects only returns for repair as a result of confirmed component removals. It does not consider returns which may have secondary (induced) failures or units when no fault can be found (unconfirmed).

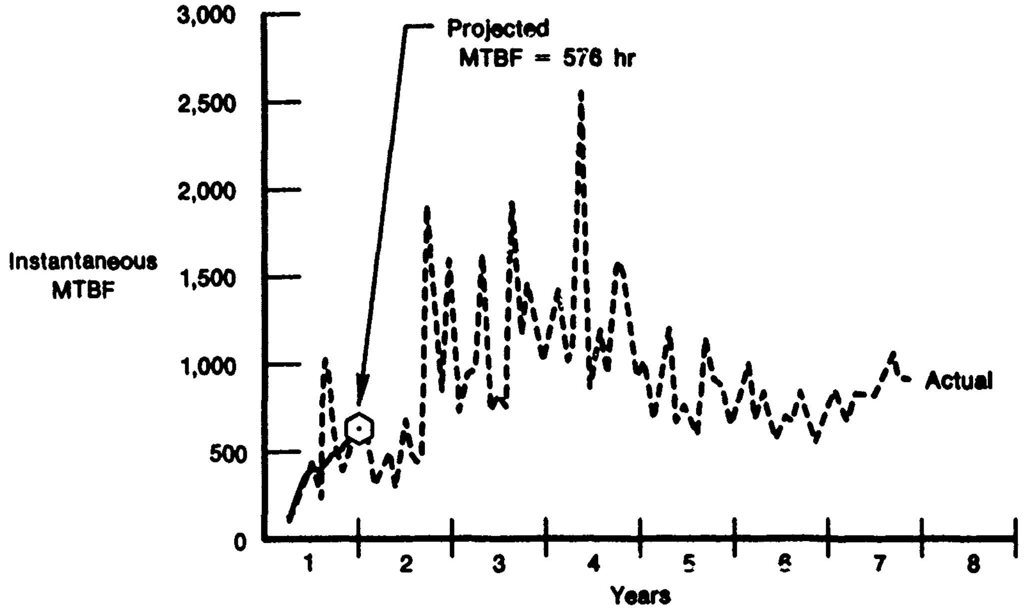

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Figure 34. Projected vs Actual Component MTBF

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SECTION VI

RESULTS AND CONCLUSIONS

A. RESULTS

1. Of all the models considered (AMSAA/Duane, Modified Duane, Cox-Lewis, Endless-Burn-In, and ARIMA) the AMSAA/Duane model was considered the best model for general use. This model was the easiest to use, could be displayed graphically, performed better than most of the other models in short- and long-term forecasting and is the most popular model in the industry.

2. There are two basic methods for estimating the parameters of the AMSAA/Duane model: (1) The first is to find, by regression analysis, the set of parameters that minimizes the sum of the squared errors (actual — predicted); (2) The second method is to find, (by likelihood equations), the set of parameters that maximizes the probability of observing the actual data. The second method, maximum likelihood estimates, was the best for forecasting future reliability.

It was determined that continuously tracked data is superior to grouped data when using the AMSAA model. The parameters can be easily and more precisely estimated, confidence limits, and goodness-of-fit tests are available, and exploratory analysis is simplified.

3. Time Series ARIMA modeling was evaluated in detail and determined to be a feasible alternative to the (NHPP) models presently being used. The cumulative failure rate (cum failures/cum time) was the best parameter to use to describe reliability and the ARIMA (0, 1, 1) (first difference moving average) was the best model form to be used.

4. A procedure has been written which illustrates, with examples, how to apply the AMSAA/Duane model and ARIMA model to electronic controls. This procedure shows how to develop to a planned curve, collect the data, analyze the data, and interpret the results.

5 The Endless-Burn-In model was determined to be an effective model for solid state electronics reliability when enough time is being obtained to see the long-term burn-in effect and the average age of the system being run is representative of the individual systems.

6. A list of references included as Appendix A on reliability growth has been compiled and categorized in Table 1 by model type (NHPP, Time Series, Deterministic, etc.) and by article content (Theory, Application, Discussion, etc.).

7. Current state-of-the-art technology for estimating reliability jumps as a result of delayed fixes was incorporated into this study.

8. The AMSAA/Duane model can be used as a tool in evaluating the reliability of components for warranty purposes; however, the accuracy of predicting the absolute number of expected returns under warranty is not high.

B. CONCLUSIONS

1. The AMSAA model is the recommended model for use in modeling military aircraft electronic engine controls.

2. Maximum Likelihood estimation is the recommended method for calculating the parameters of the AMSAA model.

3. Data should be tracked continuously on an individual and fleet basis.

4. Box-Jenkins Time Series modeling is an effective tool for modeling reliability growth when the AMSAA model is not appropriate.

5. The Endless-Burn-In model is effective in modeling solid-state electronics reliability when large amounts of time are expected and average unit age is representative of individual ages.

6. Dr. Crow's methods for estimating the effects of delayed fixes on reliability should be considered as an alternative to *data purging* (scaling failure models that have fixes in the system).

7. Reliability growth modeling can be used as a tool in assessing and projecting reliability for warranty purposes. However, the model does not provide the accuracy required to act as a sole source of information for basing warranty agreements.

8. While none of the models reviewed were effective for assessing reliability in the long term, the AMSAA/Duane model, the Endless-Burn-In model and the ARIMA model were all effective in making short-term forecasts.

APPENDIX A

LIST OF APPLICABLE DOCUMENTS

This appendix is a list of books, articles and reports reviewed for the GAPCEEC program. Most of these documents directly address reliability-growth modeling; however, a few documents address other types of statistical methodology that were used in the program to analyze data but not as a growth model. Comments have been included where appropriate. This literature has been grouped by model type and subject matter for easy reference as shown in Table A-1.

1. W. W. Abadeer, "Statistical Evaluation of Learning Factor in Reliability Studies," *IEE Trans. Reliability,* Vol R-29, Dec 1980, pp 414-415.

2. S. Abe, "Field Data Analysis Via Markov Renewal Life Models," *Proc. 1975 Annual Reliability and Maintainability Symp.*, 1975, pp 562-567.

3. H. Akaike, "Markovian Representative of Stochastic Processes and Its Application to the Analysis of Autoregressive Moving Average Processes," *Annals of the Institute of Stat. Math.*, Vol 26, 1974, pp 363-387.

   Important demonstration of the equivalence of ARMA and Markovian processes. Gives derivation of the information theoretic criterion useful in ARMA system model identification.

4. G. Almassy, "Limits of Models in Reliability Engineering," *Proc. 1979 Annual Reliability and Maintainability Symp.*, 1979, pp 364-367.

5. O. D. Anderson, *Time Series Analysis: The Box-Jenkins Approach,* Butterworths, London, 1976.

   Good introduction, with many examples, to ARIMA modeling.

6. O. D. Anderson, *Time Series Analysis and Forecasting,* London, Butterworths, 1976, pp 93-123.

7. Anonymous, "Reliability Growth Management, Testing and Modeling" *Seminar Proc., Institute of Environmental Sciences, Chesapeake Chapter,* n(83), 1978.

8. H. E. Ascher, "Evaluation of Repairable System Reliability Using Bad-as-Old Concepts," *IEEE Trans. Reliability,* Vol R-17, Jun 1968, pp 103-110.

   First paper to provide clarification and appropriate analysis using a nonhomogeneous Poisson process for studying the reliability growth of repairable mechanical systems.

9. H. E. Ascher, "Distribution-Free Estimation of System Reliability," *Proc. 1980 Annual Reliability and Maintainability Symp.*, 1980, pp 374-378.

   Exemplifies the commonalities between biostatistical survival analysis and reliability engineering analysis.

10. H. E. Ascher, "Weibull Distribution vs Weibull Process," *Proc. 1981 Annual Reliability and Maintainability Symp.*, 1981, pp 426-431.

Provides justification for avoiding the phrase "Weibull process."

11. H. Ascher, H. Feingold, "Bad as Old Analysis of System Failure Data," *Annals of Assurance Sciences, 8th Reliability and Maintainability Conf.*, 1969, pp 49-62.

12. H. Ascher, H. Feingold, "Is There Repair After Failure?," *Proc. 1978 Annual Reliability and Maintainability Symp.*, 1978, pp 190-197.

13. H. Ascher, H. Feingold, "The Aircraft Air Conditioner Data Revisited," *Proc. 1979 Annual Reliability and Maintainability Symp.*, 1979, pp 153-159.

14. N. T. J. Bailey, *The Elements of Stochastic Processes*, John Wiley & Sons, New York, NY, 1964.

15. L. J. Bain, F. T. Wright, "The Negative Binomial Process with Applications to Reliability," *J. of Quality Technology*, Vol 14, Apr 1982, pp 60-66.

Describes an alternative to the Poisson distribution for count data.

16. H. S. Balaban, "Reliability Growth Models," *J. Environmental Sciences*, Vol 21, Jan-Feb 1978, pp 11-18.

A good review of many of the current used reliability growth models.

17. R. E. Barlow, "Analysis of Retrospective Failure Data Using Computer Graphics," *Proc. 1978 Annual Reliability and Maintainability Symp.*, 1978, pp 113-116.

Promotes the use of total time on test analysis.

18. R. E. Barlow, F. Proschan, *Statistical Theory of Reliability and Life Testing*, Holt, Rinehart and Winston, New York, NY 1975.

A significant contribution to the theory of statistical reliability.

19. R. E. Barlow, E. M. Scheuer, "Reliability Growth During A Development Testing Program," *Technometrics*, Vol 8, Feb 1966, pp 53-60.

Derives the trinomial model of reliability growth.

20. D. R. Barr, "Class of General Reliability Growth Models," *Operations Research*, Vol. 18, 1970, pp 52-65.

21. D. E. Beachler, W. A. Chapman, "Reliability Proving for Commercial Products," *Proc. 1977 Annual Reliability and Maintainability Symp.*, 1977, pp 89-94.

Application of the Duane model.

22. E. F. Belot, *"A Computer Program for Estimation of Parameters of the Weibull Intensity Function and for the Cramer von Mises Goodness of Fit Test,"* Tech. Rept. 279, US Army Material Systems Analysis Activity, Aberdeen Proving Ground, MD, Jul 1979.

23. A. Bezat, F. Kreuze, *"The Endless Burn-In Growth Model for Projecting Stress-Screening Requirements,"* Internal Rept., Avionics Division, Honeywell Inc., Minneapolis, MI, 1982.

24. A. G. Bezat, L. L. Montague, "The Effect of Endless Burn-In on Reliability Growth Projections," *Proc. 1979 Annual Reliability and Maintainability Symp.*, 1979, pp 392-397.

    Presents a modified geometric curve method with supporting data for modeling reliability growth of solid-state electronic systems.

25. A. Bezat, V. Norquist, L. Montague, "Growth Modeling Improves Reliability Predictions," *Proc. 1975 Annual Reliability and Maintainability Symp.*, 1975, pp 317-322.

26. R. Billinton, M. Alam, "Effect of Restricted Repair on System Reliability Indices," *IEEE Trans. Reliability,* Vol R-27, Dec 1978 pp 376-380.

    Derives system average failure rate and average outage duration via Markov chains.

27. Z. W. Birnbaum, *On the Mathematics of Competing Risks, Vital and Health Statistics: Series 2, Data Evaluation and Methods Research,* 77, DHEW Publ., Washington, D.C. (PHS) 79-1351, 1979.

    Excellent reference on parametric and nonparametric theory and estimation techniques relating to survival analysis.

28. H. S. Blanks, "Electronic Reliability: a State of the Art Survey," *Microelectronics and Reliability,* Vol 20, 1980, pp 219-245.

    General discussion of the physics of electronic component reliability including reliability growth with extensive bibliography.

29. S. Blumenthal, J. A. Greenwood, L. H. Herbach, "The Transient Reliability Behavior of Series Systems or Superimposed Renewal Process," *Technometrics,* Vol 15, May 1973, pp 255-269.

    An interesting possible alternative to using nonhomogeneous Poisson processes to evaluate repairable system reliability growth.

30. S. Blumenthal, J. A. Greenwood, L. H. Herbach, "A Comparison of the Bad as Old and Superimposed Renewal Models," *Management Science,* Vol 23, Nov 1976, pp 280-285.

31. T. J. Boardman, M. C. Bryson, "A Review of Some Smoothing and Forecasting Techniques," *J. Quality Technology,* Vol 10, Jan 1978 pp 1-11.

32. A. J. Bonis, "Reliability Growth Curves for One Shot Devices," *Proc. 1977 Annual Reliability and Maintainability Symp.,* 1977, pp 181-185.

Uses a modified exponential function to determine growth.

33. G. E. P. Box, G. M. Jenkins, *Time Series Analysis: Forecasting and Control,* Holden-Day, San Francisco, Ca, 1976.

The basic test on ARIMA.

34. J. E. Bresenham, "Reliability Growth Models," *Tech. Rept. 74,* N(95), Defense Logistics Agency, Alexandria VA, Aug 1964.

An interesting early examination of reliability growth models.

35. J. J. Bussolini, "The Benefits of a Totally Integrated Reliability Test Program," *AGARD, Lecture Series 47 on Reliability and Avionics Systems,* 7 Rue Ancelle 92200 Nevilly Sur Seine France, 1971, pp 11-1 and 11-21.

36. J. K. Byers, D. H. Galli, "Reliability Growth Apportionment," *IEEE Trans. Reliability,* Vol R-26, Oct 1977, pp 242-244.

Demonstrates the use of a mathematical programming technique which incorporates cost data to examine reliability growth.

37. J. M. Clarke, "No-Growth Growth Curves," *Proc. 1979 Annual Reliability and Maintainability Symp.,* 1979, pp 407-410.

Stresses using engineering analysis in conjunction with statistical analysis to ensure accurate conclusions.

38. J. M. Clarke, W. P. Cougan, "A Recent Real Life Case History," *Proc. 1978 Annual Reliability and Maintainability Symp.,* 1978, pp 231-242.

Duane model applied to electronics.

39. J. P. Cleary, H. Leavenbach, *The Professional Forecaster,* Lifetime Learning Publications, Belmont, CA, 1982, pp 263-282.

40. E. O. Codier, "Reliability Growth in Real Life," *Proc. 1968 Annual Symposium on Reliability,* 1968, pp 458-469.

Promotes use of the Duane model.

41. W. J. Corcoran, H. Weingarten, P. W. Zehna, "Estimating Reliability After Corrective Actons," *Management Science,* Vol 10, Jul 1964, pp 786-795.

Demonstrates a method for purging data and estimating current reliability after corrective action based on the binomial distribution.

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42. W. J. Corcoran, R. R. Read, "Study of the Reliability Growth Implications of Subsystem vs Full Assembly Testing," United Technologies Research Center, *UTC 2140-FR*, United Aircraft Corporation Sunnyvale, Calif, Nov 1968.

43. D. R. Cox, "Regression Models and Life Tables," *J. Royal Stat. Soc.*, Series B, Vol 34, 1972, pp 187-220.

A milestone in survival data analysis methods.

44. D. R. Cox, P. A. W. Lewis, *The Statistical Analysis of Series of Events*, Chapman and Hall, London, 1966.

The foundational text on analyzing stochastic point processes.

45. T. D. Cox, J. Keely, "Reliability Growth Management of SATCOM Terminals," *Proc. 1976 Annual Reliability and Maintainability Symp.*, 1976, pp 218-221.

Comparison of Duane's model to Crow's nonhomogeneous Poisson process model using electronic system failure data.

46. W. P. Cougan, W. G. Kindig, "A Real Life MTBF Program for a Deployed Radar," *Proc. 1979 Annual Reliability and Maintainability Symp.*, 1979, pp 212-127.

Application of Duane's model to electronic system reliability growth.

47. L. H. Crow, *Interim Note No. 27 On Reliability Growth Modeling*, U. S. Army Material Systems Analysis Activity, Aberdeen Proving Ground, MD, Jan 1974.

48. L. H. Crow, "Reliability Analysis for Complex Repairable Systems," *Soc. Industrial and Applied Mathematics, Reliability and Biometry, Proceedings of Statistical Analysis of Life Length*, Jul 1974, pp 379-410

Provides derivation of maximum likelihood estimators, hypotheses testing, confidence intervals, and goodness-of-fit procedures for modeling repairable system reliability growth via a nonhomogeneous Poisson process with a Weibull intensity function.

49. L. H. Crow, "On Tracking Reliability Growth," *Proc. 1975 Annual Reliability and Maintainability Symp.*, 1975, pp 438-443.

Application of Crow's model to missile system data.

50. L. H. Crow, *Confidence Interval Procedures for Reliability Growth Analysis*, U.S. Army Material Systems Analysis Activity, Tech. Rept. 197, Aberdeen Proving Ground, MD, Jun 1977.

51. L. H. Crow, "Confidence Interval Procedures for the Weibull Process With Applications to Reliability Growth," *Technometrics*, Vol 24, Feb 1982, pp 67-72.

52. L. H. Crow, "An Improved Methodology for Reliability Growth Projections," *Technical Report 357,* U.S. Army Material Systems Analysis Activity, Aberdeen, MD, June 1982.

53. F. Croxton, D. Cowdwn, *General Applied Statistics,* Prentice-Hall, Englewood Cliffs, NJ, Chap 13, pp 282-319, 1956.

Details manual computing methods to estimate the parameters of the Gompertz and logistic functions.

54. R. C. Dahiya, "Estimation of Reliability After Corrective Action," *IEEE Trans. Reliability,* Vol R-26 Dec 1977, pp 348-351.

Gives asymptotic distributions for estimators.

55. B. S. Dhillon, "Mechanical Reliability Interference Theory Models," *Proc. 1980 Annual Reliability and Maintainability Symp.* 1980, pp 462-467.

Reliability dealt with in terms of stress and strength concepts. Large bibliography.

56. B. S. Dhillon, "New Hazard Rate Functions," *Microelectronics and Reliability,* Vol 18, 1979, pp 531-532.

57. B. S. Dhillon, "Reliability Growth: A Survey," *Microelectronics and Reliability,* Vol 20, 1980, pp 743-751.

Very large bibliography.

58. B. S. Dhillon, "Life Distribution," *IEEE Trans. Reliability,* Vol R-30, Dec 1981, pp 457-460

Presents two new distributions.

59. Y. M. I. Dirickx, K. P. Kistner, "Reliability of a Repairable System with Redundant units and Preventive Maintenance," *IEEE Trans. Reliability,* Vol R-28, Jun 1979, pp 170-171.

60. D. A. Dobbins, "The Dangers of Relying on Industry as a Partner in Material Development," *Test,* Aug/Sept 1980, pp 8-13.

61. J. Donelson III, "Cost Model for Testing Program Based on Nonhomogeneous Poisson Failure Model, " *IEEE Trans. Reliability,* Vol R-26, Aug 1977, pp 189-194.

62. J. T. Duane, "Learning Curve Approach to Reliability Monitoring," *IEEE Trans. Aerospace,* Vol, 2 Apr 1964, pp 563-566.

Duane's presentation of his model.

63. D. R. Earles, "Reliability Growth Prediction During the Initial Design Analysis," *Proc. IEEE 7th Natl. Symp. on Reliability and Quality Control,* 1961, pp 380-393.

Early empirical evaluation of reliability growth.

64. R. G. Easterling, "Approximate Confidence Limits for System Reliability," *J. Amer. Stat. Assoc.*, Vol 67, Mar 1972, pp 220-222.

65. J. D. Ekings, R. L. Sweetland, "Burn-in Forever? There Must be a Better Way," *Proc. 1978 Annual Reliability and Maintainability Symp.*, 1978, pp 286-293.

66. E. A. Elsayed, A. Zebib, "A Repairable Multistate Device," *IEEE Trans. Reliability*, Vol R-28, Apr 1979, pp 81-82.

67. R. A. Evans, "Models for Reliability of Repaired Equipment," *IEEE Trans. Reliability*, Vol 19, Nov 1970, p 127.

68. B. V. Faden, D. Rodbard, *EXPFIT: Exponential Data Processing*, Biomedical Computing Technology Information Center Computer Code Collection, Vanderbilt Medical Center, TN., 1975.

FORTRAN code to fit one, two and three component exponential functions.

69. K. W. Fertig, V. K. Murphy, "Models for Reliability Growth During Burn-in: Theory and Applications," *Proc. 1978 Annual Reliability and Maintainability Symp.*, 1978, pp 504-509.

Presents three new Bayesian models for electronic systems.

70. R. W. Fink, "Screening for Reliability Growth," *Proc. 1971 Annual Symp. on Reliability*, 1971, pp 316-320.

71. J. M. Finkelstein, "Starting and Limiting Values for Reliability Growth," *IEEE Trans. Reliability*, Vol R-28, Jun 1979, pp 111-114.

Expands Duane's/Crow's model(s) to three parameters to produce mathematical limits of growth.

72. J. M. Finkelstein, "Confidence Bounds on the Parameters of the Weibull Process," *Technometrics*, Vol 18, No. 1, 1976.

73. E. L. Frome, M. H. Kutner, J. J. Beauchamp, "Regression Analysis of Poisson-Distributed Data," *J. Amer. Stat. Assoc.*, Vol 68, Dec 1973, pp 935-940.

74. W. A. Fuller, *Introduction to Statistical Time Series*, John Wiley & Sons, New York, NY 1976, pp 372-373.

75. A. E. Gelfand, "An Estimation Problem with Poisson Processes," *Austral. J. Stat.*, Vol 23, Aug 1981, pp 224-231.

Reviews Poisson process intensity function estimation.

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76. W. R. Gjertsen, "Some Steady State Queueing System Macros," *Statistical Analysis System Users Group Internatl. Proc. of the 6th Ann. Conf.*, SAS Institute, Cary, NC, 1981, pp 304-308.

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77. A. L. Goel, K. Okumoto, "Time Dependent Error-Detection Rate Model for Software Reliability and Other Performance Measures," *IEEE Trans. Reliability*, Vol R-28, Aug 1979, pp 206-211.

Presents various intensity functions for nonhomogeneous Poisson processes applicable to general reliability growth.

78. O. Gokcek, I. Bazovsky Sr., G. L. Crellin, "Markov Analysis of Nuclear Plant Failure Dependencies," *Proc. 1979 Annual Reliability and Maintainability Symp.*, 1979, pp 104-109.

79. C. W. Green, "PMOS Dynamic RAM Reliability — a Case Study," *Proc. 1979 Annual Reliability Physics Symp.*, 1979, pp 213-219.

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Daune model application.

83. A. J. Gross, "The Application of Exponential Smoothing to Reliability Assessment," *Technometrics*, Vol 13, Nov 1971, pp 877-883.

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Excellent survey.

85. A. J. Gross, M. Kamins, "Reliability Assessment in the Presence of Reliability Growth," *Proc. 1968 IEEE Symp. on Reliability*, 1968, pp 406-426.

86. E. J. Hannan, "The Identification and Parameterization of ARMAX and State Space Forms," *Econometrica*, Vol 44, Jul 1976, pp 713-722.

87. J. A. Hartigan, *Clustering Algorithms*, John Wiley & Sons, New York, 1975.

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A Bayesian approach to the estimation of the intensity function of Crow's model.

89. G. W. Hill, D. Woodworth, "Automatic Box-Jenkins Forecasting," *J. Operational Research Soc.,* Vol 31, 1980, pp 413-422.

90. R. G. Hoffman, "A Simulation Study of Autoregressive and Moving Average Time Series Estimators for Very Short Series," *Proc. Stat. Computing Section,* Amer. Stat. Assoc. 1981, pp 278-283.

Evaluates biases in small sample ARIMA forecasting.

91. M. Hollander, "Testing Whether More Failures Occur Later," *Proc. 1978 Annual Reliability and Maintainability Symp.,* 1978, pp 103-106.

Presents tests for "new better than used" for use on a nonhomogeneous Poisson process.

92. M. Hollander, D. A. Wolfe, *Nonparametric Statistical Methods,* John Wiley & Sons Inc., New York, NY, p 138.

93. P. T. Holmes, D. E. Huffman, *Estimation of Selected Parameters in Nonhomogeneous Poisson Processes* Rept. N57, Dept. of Math. Sci., Clemson Univ., Jan 1975.

Mathematically abstract paper.

94. P. T. Holmes, D. E. Huffman, *Estimation of the Intensity and Mean Value Functions of a Nonhomogeneous Poisson Process,* Rept. N57, Dept. of Math. Sci., Clemson Univ., Jun 1975.

95. P. T. Holmes, D. E. Huffman, *Estimation of the Peak Rate and Related Parameters in Nonhomogeneous Poisson Processes,* Rept. N59, Dept. of Math. Sci., Clemson Univ., Jun 1975.

96. S. C. Hora, "A Screening Test for the Poisson Process," *Decision Sciences,* Vol 9, Jul 1978, pp 414-420.

Provides a test for the realization of a homogeneous Poisson process.

97. E. T. Ireton, "Evolutionary Growth of Helicopter Maintainability, Reliability, and Safety," *Annals of Assurance Sciences,* 1968, pp 167-176.

98. P. A. Jacobs, P. A. W. Lewis, "Discrete Time Series Generated by Mixtures I: Correlational and Runs Properties, " *J. Royal Stat. Soc.,* Vol 40, 1978, pp 94-105.

Highly mathematical and theoretical.

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99. P. A. Jacobs, P. A. W. Lewis, "Discrete Time Series Generated by Mixtures II: Asympototic Properties," *J. Royal Stat. Soc.*, Vol 40, 1978, pp 222-228.

100. P. A. Jacobs, P. A. W. Lewis, *Discrete Time Series Generated by Mixtures III: Autoregressive Processes (DAR(p))*, Naval Postgraduate School, Monterey, CA, NPS55-78-022, 1978.

101. P. A. Jacobs, P. A. W. Lewis, *Stationary Discrete Autoregressive-Moving Average Time Series Generated by Mixtures*, Naval Postgraduate School, Monterey, CA, NPS55-82-003, 1982.

102. F. E. Jaumot, "Reliability, an Industry Viewpoint," *Proc. Institute of Environmental Sciences*, Sep 1981, pp 19-25.

103. T. Jayachandran, L. R. Moore, "A Comparison of Reliability Growth Models," *IEEE Trans. Reliability*, Vol R-25, Apr 1976, pp 49-51.

Comparative simulation study of Duane's model, Weiss' model, Chernoff and Wood's model, and Wolman's model.

104. W. S. Jewell, *Reliability Growth as an Artifact of Renewel Testing*, Operations Research Center, Univ. Calif. Berkeley, CA, Report ORC 78-9, Jun 1978.

A Bayesian examination of reliability growth.

105. W. S. Jewell, *Stochastically-Ordered Parameters in Bayesian Prediction*, Operations Research Center, Univ. Calif., Berkeley, CA, Report ORC 79-12, Oct 1979.

A study of the concept of reliability growth by stages.

106. J. D. Johnson, L. T. Stewart, "Failure Prediction From Interval Data," *Proc. 1971 Annual Symp. on Reliability*, 1971, pp 356-361.

107. K. C. Kapur, L. R. Lamberson, *Reliability in Engineering Design*, John Wiley & Sons, New York, 1977.

108. E. L. Kaplan, P. Meier, "Nonparametric Estimation from Incomplete Observations," *J. Amer. Stat. Assoc.*, Vol 53, Jun 1958, pp 457-481.

109. G. Kasouf, "The Fallacious Reliability Demonstration Test," *Proc. 1979 Annual Reliability and Maintainability Symp.*, 1979, pp 374-380.

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TABLE A-1. LITERATURE CATEGORIZED BY MODEL TYPE AND CONTENT

Model Type	Subject Matter: Theory	Subject Matter: Methods	Electronic Application	Mechanical Application	General Discussion
Deterministic	1, 15, 16, 17, 18, 19, 27, 34, 36, 41, 54, 55, 56, 58, 64, 73, 84, 115 145, 172, 177, 178, 195, 200, 204, 208, 210, 212	1, 9, 15, 18 23, 24, 25, 27, 32, 36, 40, 41, 53, 64, 68, 73, 76, 145, 177, 178, 189, 195, 205, 208, 209, 210	23, 24, 25, 38, 46, 133, 142, 153, 174	9, 133, 177, 190, 209	4, 16, 18, 21, 23, 24, 25, 32, 34, 37, 40, 41, 42, 45, 46, 53, 57, 62, 63, 65, 70, 71, 81, 82, 102, 115, 142, 145, 162, 163, 174, 182, 183, 185, 197, 203, 204, 209, 214
Renewal Process	14, 18, 29, 30, 44, 59, 175, 176	29, 30, 41, 59, 175, 176			44
Markov Process	2, 3, 14, 18, 26, 34, 44, 59, 66, 175, 176, 184, 188	2, 14, 26, 78, 175, 176		2, 78	14, 34, 44
Nonhomogeneous Poisson Process	8, 11, 14, 16, 18, 44, 48, 49, 50, 51, 52, 61, 71, 72, 75, 77, 124, 145, 149, 164, 195, 196, 211	11, 12, 13, 14, 48, 49, 50, 51, 52, 61, 71, 72, 75, 77, 124 145, 149, 195, 196	133	11, 12, 13, 48, 49, 61, 133	8, 10, 11, 13, 16, 44, 45, 48, 60, 67, 145, 186, 197
Semi-Markov Process	167, 175, 176, 189	9, 167, 175 176, 189		9	
Time Series Process	3, 5, 33, 74, 83	5, 33, 39, 74, 83, 191 192, 193		191, 192, 193	5, 31, 33
Bayesian	69, 83, 88, 104, 105, 134, 165, 194	69, 83, 88, 134, 165, 194			

APPENDIX B

Part 1 — Criteria for Reliability-Growth Model Selection as Established in Task 200 (July 1982)

1. Small sample forecasting — The reliability-growth can be predicted for small samples. (5)*

2. Tracking ability — The reliability-growth can be tracked statistically, i. e., the actual growth follows the predicted growth curves. (6)*

3. Realistic final results — The model will predict final results that are comparable with observed data. (2)*

4. Physical interpretation — The model will be based on engineering concepts as well as statistical concepts. (7, 8)*

5. Graphic display of results — Some graphic techniques for displaying data and model will be available. (9)*

6. Adaptability — The model will be able to handle development and field data, to capture the check-mark phenomenon, and to handle repairable systems. (10, 11)*

7. Valid goodness-to-fit tests — Some statistically valid goodness-to-fit tests will be available for checking the reliability-growth models. (12)*

8. Realistic asymptotic values — The reliability-growth model will produce realistic asymptotic values with the increased time or sample size. (14)*

9. Efficient/consistent parameter estimation — The reliability-growth model will provide means of accurate and consistent estimation of various parameters . (15)*

10. Confidence limits for parameters — Methodology will be available to provide interval estimates for the parameters of the reliability-growth model. (13)*

11. Test for trends — A test will be available to distinguish between random variation and actual reliability growth. (13)*

Part 2 — Statements Describing Criterion for GAPCEEC Model Comparison

1. The model satisfactorily generates 12-month forecasts.**

2. The model satisfactorily forecasts reliability at maturity from development data.

3. The model forecasts field entry reliability within ±10% with adequate lead time for review and action.**

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*Corresponding statement number in Part 2.
**Added statements to include work done on long-term forecasting.

4. The model forecasts mature life reliability within $\pm 10\%$ with adequate lead time for review and action.**

5. The model generates useful forecasts from small samples.

6. The model is helpful in evaluating reliability growth management strategy.

7. The models parameters can be equated directly to reliability engineering concepts.

8. The model has meaning in a physical environment.

9. The model parameters can be estimated easily with graphic displays.

10. The model can adapt quickly to changes in data trends.

11. The model is applicable to several types of components (electronic, mechanical, etc.).

12. One or more tests are available that measure the goodness-to-fit between the model and the data.

13. One or more tests are available that measure the contribution of the parameters to the model's fit of the data.

14. The model contains a limiting parameter that limits reliability forecasts to realistic values.

15. Efficient, consistent estimation of the models is done easily.

APPENDIX C

MAXIMUM LIKELIHOOD ESTIMATION OF PARAMETERS OF RELIABILITY GROWTH MODELS

The mathematics of the maximum likelihood estimation of parameters of three reliability growth models is presented in this appendix (Duane, Cox-Lewis and Modified Duane Models). The Maximum Likelihood (ML) estimators of the Duane Model has been used in this study.

Maximum Likelihood Estimation of Parameters of Reliability Growth Models

A. BACKGROUND/GENERAL INFORMATION

Consider a nonhomogeneous Poisson process with intensity function ρ(t). The mean value function (expected number of failures by time t) is

$$M(t) = \int_0^t \rho(x)\,dx.$$

The probability of j failures by time t (i.e., the interval o,t) is

$$P\,(j \text{ Failures } \varepsilon\,(o,t)) = \frac{M(t)^j\, e^{\,M(t)}}{j\,!}$$

If there has been recorded times for k intervals, i = 1, . . ., k with n_i failures in interval i, etc., then the probability of this event (n_i failures in interval i) is as follows:

$$P(n_i \text{ failures } \varepsilon\,(t_{i-1}, t_i)) = \frac{[M(t_i) - M(t_{i-1})]^{n_i}\, e^{-[M(t_i) - M(t_{i-1})]}}{n_i\,!}$$

The probability of all intervals having the observed number of failures is the product of the probabilities associated with each interval (assuming independence).

$$P(n_1 \text{ failures } \varepsilon\,(o,t_1) \text{ and } n_2 \text{ failures } \varepsilon\,(t_1, t_2)\cdot\;\cdot\;\cdot \text{ and } n_k \text{ failure } \varepsilon\,(t_{k-1}, t_k)) = \pi_{i=1}^{k}\, P(n_i\, \varepsilon\,(t_{i-1}, t_i)).$$

This is the likelihood of observing exactly what has happened to date, and is called the likelihood function (LF).

the (ML) Maximum likelihood estimates of the parameters in ρ (t) or M(t)) are those parameter values that maximize the likelihood function.

$$LF = \pi_{i=1}^{k}\, P(n_i\, \varepsilon\,(t_{i-1}, t_i)) = \pi_{i=1}^{k} \frac{[M(t_i) - M(t_{i-1})]^{n_i}}{n_i\,!}\, e^{\;[M(t_i)\; - \;M(t_{i-1})]}$$

It is sufficient to maximize the $\log_e$ (LF). (See "Introduction to the Theory of Statistics," Mood, Graybill, and Boes, Third Edition, page 279.)

$$\log_e (LF) = \sum_{i=1}^{k} n_i \log_e (M(t_i) - M(t_{i-1}))\,(M(t_i) - M(t_{i-1})) - \log_e (n_i)$$

Since $\log_e(n_i)$ does not depend on the parameters of M(t), it is sufficient to maximize $\log_e(L')$, where $\log_e(L')$ is as follows:

$$\log_e(L') = \sum_{i=1}^{k} n_i \log_e(M(t_i) - M(t_{i-1})) - (M(t_i) - M(t_{i-1}))$$

which can be simplified to the following:

$$\log_e(L') = -[M(t_k) - M(t_o)] + \sum_{i=1}^{k} n_i \log_e(M(t_i) - M(t_{i-1}))$$

B. THE LIKELIHOOD EQUATIONS

To find the parameters of M(t) that maximize $\log_e(L')$, take the derivative of $\log_e(L')$ with respect to each parameter, set the equation equal to zero, and solve. If M(t) has parameters Θ_1, $\Theta_2, \ldots \Theta_M$, then the set of equations would be as follows:

$$\frac{\delta \log_e(L')}{\delta\Theta_1} = \frac{-\delta M(t_k)}{\delta\Theta_1} + \frac{\delta M(t_o)}{\delta\Theta_1} + \sum_{i=1}^{k} \frac{n_i\left[\frac{\delta M_{(t_i)}}{\delta\Theta_i} - \frac{\delta M(t_{i-1})}{\delta\Theta_1}\right]}{M(t_i) - M(t_{i-1})}$$

$$\vdots$$

$$\frac{\delta \log_e(L')}{\delta\Theta_M} = \frac{-\delta M(t_k)}{\delta\Theta_M} + \frac{\delta M(t_o)}{\delta\Theta_M} + \sum_{i=1}^{k} \frac{n_i\left[\frac{\delta M(t_i)}{\delta\Theta_M} - \frac{\delta M(t_{i-1})}{\delta\Theta_M}\right]}{M(t_i) - M(t_{i-1})}$$

Note that $M(t_o) = 0$ for all models, i.e., there are 0 failures by time $t = 0$.

C. THE DUANE MODEL (SEE MIL-HDBK-189)

$\rho(t) = \lambda\beta t^{\beta-1.}$ λ and β are parameters, t is cumulative time.

$M(t) = \lambda t^{\beta}$

$\frac{\delta M(t)}{\delta\lambda} = t^{\beta}$ and $\frac{\delta M(t)}{\delta\beta} = \lambda t^{\beta}\log_e(t)$.

$$\frac{\delta \log_e(L')}{\delta\lambda} = -t_k^{\beta} + \sum_{i=1}^{k} \frac{n_i(t_i^{\beta} - t_{i-1}^{\beta})}{\lambda(t_i^{\beta} - t_{i-1}^{\beta})} = -t_k^{\beta} + 1/\lambda \sum_{i=1}^{k} n_i = 0.0.$$

$$\text{if } \sum_{i=1}^{k} n_i = N \text{ then } \frac{\delta \log_e(L')}{\delta\lambda} = -t_k^{\beta} + \frac{N}{\lambda} = 0.0$$

$$\frac{\delta \log_e(L')}{\delta\beta} = -\lambda t_k^{\beta}\log_e(t_k) + \sum_{i=1}^{k} \frac{n_i[\lambda t_i^{\beta}\log_e(t_i) - \lambda t_{i-1}^{\beta}\log_e(t_{i-1})]}{\lambda t_i^{\beta} - \lambda t_{i-1}^{\beta}}$$

$$= -\lambda t_k^{\beta}\log_e(t_k) + \sum_{i=1}^{k} \frac{n_i[t_i^{\beta}\log_e(t_i) - t_{i-1}^{\beta}\log_e(t_{i-1})]}{(t_i^{\beta} - t_{i-1}^{\beta})} = 0.0.$$

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Since $-t_k^\beta + \frac{N}{\lambda} = 0, \lambda = \frac{N}{t_k^\beta}$. Substituting above,

$$\frac{\delta \log_e(L')}{\delta\beta} = \frac{-N}{t_k^\beta} \cdot t_k^\beta \log_e(t_k) + \sum_{i=1}^{k} \frac{n_i\,[t_i^\beta \log_e(t_i) - t_{i-1}^\beta \log_e(t_{i-1})]}{t_i^\beta - t_{i-1}^\beta} = 0.0$$

$$\frac{\delta \log_e(L')}{\delta\beta} = -N \cdot \log_e(t_k) + \sum_{i=1}^{k} \frac{n_i\,[t_i^\beta \log_e(t_i) - t_{i-1}^\beta \log_e(t_{i-1})]}{t_i^\beta - t_{i-1}^\beta} = 0.0$$

Which can be solved numerically (with a computer) for $\hat{\beta} \approx \beta$. then, using β, $\hat{\lambda} \approx \lambda$ can be obtained from the equation

$$\hat{\lambda} = N/t_k^{\hat{\beta}}.$$

Therefore, the MLE's for the Duane Model are β and λ.

D. COX-LEWIS MODEL

$$\rho(t) = e^{\gamma + \alpha t} \text{ and } M(t) = \frac{1}{\alpha} e^{\gamma} (e^{\alpha t} - 1).$$

$$\frac{\delta M(t)}{\delta\gamma} = 1/\alpha\, e^{\gamma} (e^{\alpha t} - 1).$$

$$\frac{\delta M(t)}{\delta\alpha} = 1/\alpha\, e^{\gamma} [e^{\alpha t} (t - 1/\alpha) + 1/\alpha].$$

$$\frac{\delta \log_e(L')}{\delta\gamma} = -1/\alpha e^{\gamma}(e^{\alpha t_k} - 1.) + \sum_{i=1}^{k} \frac{n_i\,[1/\alpha e^{\gamma}(e^{\alpha t_i} - 1) - 1/\alpha e^{\gamma}(e^{\alpha t_{i-1}} - 1)]}{1/\alpha e^{\gamma} (e^{\alpha t_i} - 1) - 1/\alpha e^{\gamma}(e^{\alpha t_{i-1}} - 1)}$$

$$= -1/\alpha e^{\gamma}(e^{\alpha t_k} - 1) + N = 0.$$

$$\frac{\delta \log_e(L')}{\delta\alpha} = 1/\alpha e^{\gamma} [e^{\alpha t_k} (t_k - 1/\alpha) + 1/\alpha] +$$

$$\sum_{i=1}^{k} \frac{n_i\;\; 1/\alpha e^{\gamma} [e^{\alpha t_i}(t_i - 1/\alpha) + 1/\alpha] - 1/\alpha e^{\gamma} [e^{\alpha t_{i-1}}(t_{i-1} - 1/\alpha) + 1/\alpha]\;]}{1/\alpha e^{\gamma} (e^{\alpha t_i} - 1) - 1/\alpha e^{\gamma} (e^{\alpha t_{i-1}} - 1)}$$

or,

$$\frac{\delta \log_e(L')}{\delta\alpha} = -1/\alpha\, e^{\gamma} [e^{\alpha t_k}(t_k - 1/\alpha) + 1/\alpha] +$$

$$\sum_{i=1}^{k} \frac{n_i\,[(t_i - 1/\alpha)\, e^{\alpha t_i} - (t_{i-1} - 1/\alpha)\, e^{\alpha t_{i-1}}]}{e^{\alpha t_i} - e^{\alpha t_{i-1}}} = 0.0$$

Since $\frac{\delta \log_e(L')}{\delta\gamma} = -1/\alpha\, e^{\gamma} (e^{\alpha t_k} - 1) + N = 0,$

$$-1/\alpha e^{\gamma} = -N/(e^{\alpha t_k} - 1).$$

Therefore, α can be estimated by the following equation:

$$\frac{-N}{(e^{\alpha t_k} - 1)}[e^{\alpha t_k}(t_k - 1/\alpha) + 1/\alpha] + \sum_{i=1}^{k} n_i \left[\frac{t_i e^{\alpha t_i} - t_{i-1} e^{\alpha t_{i-1}}}{e^{\alpha t_i} - e^{\alpha t_{i-1}}} - 1/\alpha \right] = 0.0.$$

Then estimate γ by

$$\hat{\gamma} = \log_e(N) + \log_e(\hat{\alpha}) - \log_e(e^{\hat{\alpha} t_k} - 1).$$

Thus, the MLE's for the Cox-Lewis Model are α and γ. It should be noted that there are numerical problems in estimating α and γ.

E. THE MODIFIED DUANE MODEL

$\rho(t) = \lambda\beta t^{\beta-1} + \Theta$. λ, β, and Θ are parameters, t is cumulative test time.

$M(t) = \lambda t^{\beta} + \Theta t.$

$$\frac{\delta M(t)}{\delta\Theta} = t, \frac{\delta M(t)}{\delta\lambda} = t^{\beta} \text{ and } \frac{\delta M(t)}{\delta\beta} = \lambda t^{\beta}\log_e(t)$$

$$\frac{\delta \log_e(L')}{\delta\Theta} = -t_k + \sum_{i=1}^{k} \frac{n_i(t_i - t_{i-1})}{\lambda t_i^{\beta} + \Theta t_i - \lambda t_{i-1}^{\beta} - \Theta t_{i-1}} = 0.0$$

$$\frac{\delta \log_e(L')}{\delta\lambda} = -t_k^{\beta} + \sum_{i=1}^{k} \frac{n_i(t_i^{\beta} - t_{i-1}^{\beta})}{\lambda t_i^{\beta} + \Theta t_i - \lambda t_{i-1}^{\beta} - \Theta t_i} = 0.0$$

$$\frac{\delta \log_e(L')}{\delta\beta} = -t_k^{\beta}\log_e(t_k) + \sum_{i=1}^{k} \frac{n_i[\lambda t_i^{\beta}\log_e(t_i) - \lambda t_{i-1}^{\beta}\log_e(t_{i-1})]}{\lambda t_i^{\beta} + \Theta t_i - \lambda t_{i-1}^{\beta} - \Theta t_{i-1}} = 0.0.$$

Estimates of β, Θ and λ, namely $\hat{\beta}$, $\hat{\Theta}$, and $\hat{\lambda}$ can be obtained by solving the three equations above. This is a difficult problem since all three equations are nonlinear with respect to the parameters.

APPENDIX D

TASK 300 — PROCEDURE

A. PRELIMINARY ANALYSIS/BACKGROUND RESEARCH

1. Introduction

This reliability-growth modeling procedure has been developed to assist the user in applying reliability-growth modeling technique to the development of electronic engine controls (EEC). The primary reference used in developing this procedure was MIL-HDBK-189, "Reliability Growth Management." This procedure is intended to serve as a guide for applying the AMSAA/Duane model (Army Material Systems Analysis Activity) to a development program and individual tailoring should be expected. It is assumed that reliability has been a goal of the development program and was considered in the initial design of the system and in the development of the initial test plan. A typical development plan is illustrated in Figure D-1.

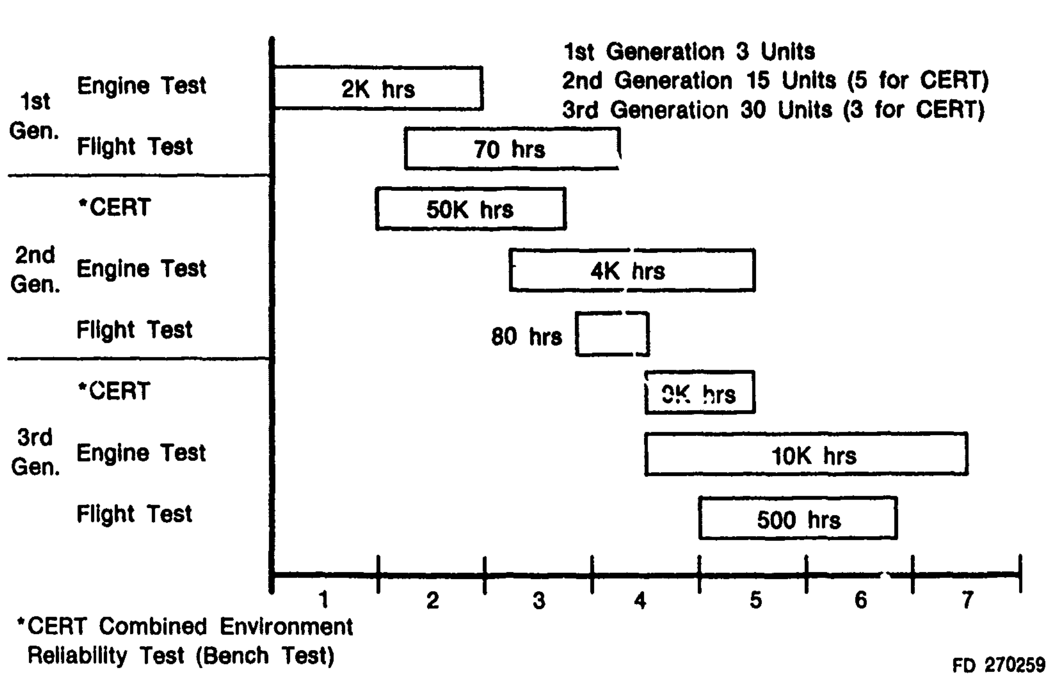

Figure D-1. Typical Development Program

2. Review of the Program Plan

It is important to understand exactly what the reliability goals are and if they are obtainable. The final MTBF (mean time between failures), will be needed for constructing the planned growth rate.

The development program should be reviewed in detail to define major test phases. These test phases will be modeled separately. A test phase is a distinct period of time during development when the system is subjected to development testing and subsequent fixes are made. Test phases are usually aligned with management goals and objectives. A test phase would usually end at a point when the program is assessed and delayed fixes incorporated.

For example, the program plan shown previously in Figure D-1 could initially be separated into phases as indicated in Figure D-2. Phase 1 consists of the initial design configuration (generation 1) tested on engine test during the first year. This phase should also include additional engine and flight test data accumulated on generation 1 controls (years 3 and 4). After the first year, a second generation control system with delayed fixes from the first generation program will go to test. This test is designed to simulate actual operating environment and the primary purpose of the test is to develop and assess the reliability of the control. This combined environment reliability test (CERT) is considered Phase 2. Phase 3 corresponds to engine and flight test data from generation 2 units. Phase 2 and 3 data can possibly be pooled since they are all from generation 2 units but this should be done only after reviewing the data. Phases 4 and 5 are the same as 2 and 3, but for generation 3 units.

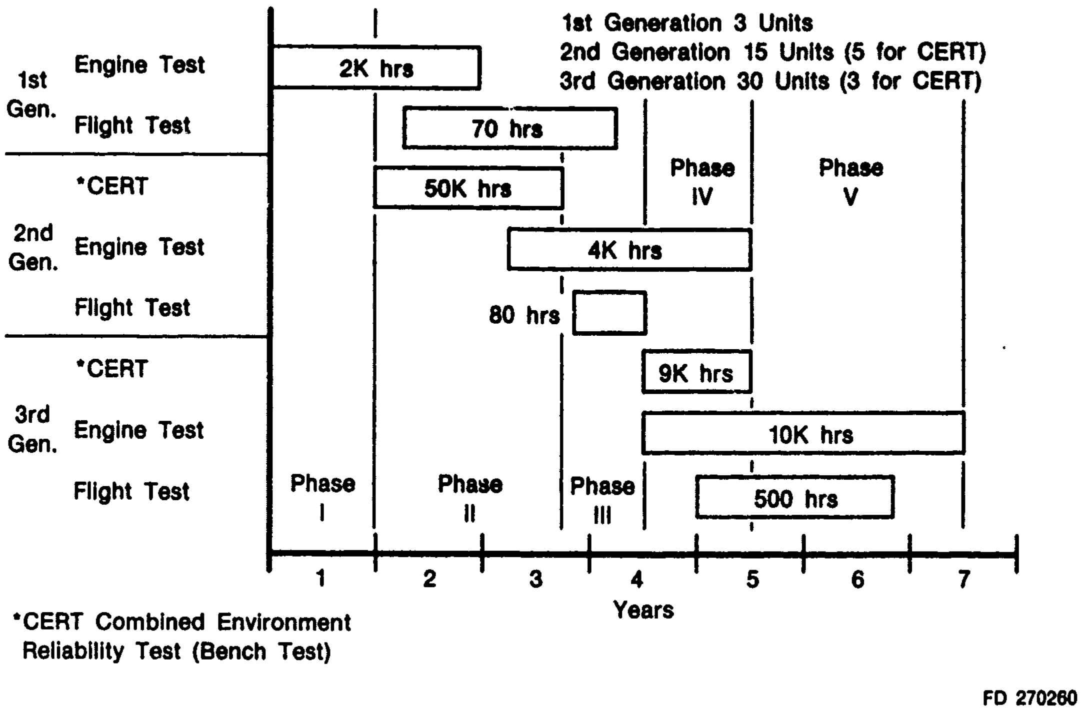

Figure D-2. Typical Development Program With Test Phases

Each of the defined test phases should be reviewed to determine their expected and planned effect on reliability. For example, phase 2 (CERT on generation 2 units should show significant reliability growth because of the ability of this test to accumulate large amounts of time (50,000 hours) within a short calendar period (1½ years).

3. Research Historical Information

After reviewing the test plan and defining test phases, the background information on this or similar systems should be compiled. Any growth modeling information on this or similar systems should be reviewed. A review of the past growth rate experience and initial reliability are also important when considering the feasibility of a planned growth curve. A review of the physical makeup of the system would also be helpful. For example, a mechanical system may be expected to experience more early wearout failure modes than an electronic system.

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4. Assessment Capabilities

Computer power, graphics capabilities, computer software available and data systems are important considerations in model selection and data collection. For example, Box-Jenkins Time Series Models require more sophisticated software than the AMSAA/Duane Model.

5. Data Constraints

A data collection system should be set up so that data could be tracked continuously. While only cumulative fleet time at the time of failure is required for the AMSAA model parameters to be estimated, individual component times and the cause of the failure are required when calculating the influence of delayed fixes on reliability. Therefore, the following data should be collected:

(1) Serial number of the failed unit
(2) Date the failure occurred
(3) Time on the failed part
(4) Time on the fleet
(5) Cause of the failure.

EEC CERT data is an example of the type of information required.

Data is usually tracked in one of two ways. If the exact amount of time on the system and fleet is known for each failure, the system is being tracked continuously. If the test is stopped after a prespecified amount of time, the test is time truncated. For example, the CERT test in Figure 1 is planned for 50,000 hours. This is a time truncated continuously tracked test. Sometimes data is gathered only at discrete intervals. For example, time and failures may be reported by month, typical of field data, or the system may be such that failures are found at inspections. This data is group data. Continuously tracked data is always preferred when available.

B. MODEL SELECTION

There are many models available for use in tracking and predicting reliability-growth (MIL-HDBK-189, pp. 108-120). Of these models, the AMSAA/Duane model, Box-Jenkins Time Series Modeling and the Endless-Burn-In model are good candidates for modeling the reliability growth of electronic engine controls.

1. The AMSAA/Duane Model

The AMSAA/Duane Modeling procedure developed by AMSAA personnel, Aberdeen Proving Grounds, Maryland, is the model that is generally recommended for use in modeling reliability-growth of an electronic engine control over a development program.

The Duane model is used for planning the general reliability growth across the entire development program (all test phases). This is a management tool that is used to determine the feasibility of the program and for comparing reliability at various points throughout the program.

The AMSAA model is a statistical tool useful for short term projections of reliability, assessment of current reliability and comparison of present levels of reliability to the reliability goals.

a) Model Description

The Duane model is a curve of cumulative failure rate as a function of cumulative test time in the development program.

$C(t) = \gamma t^{\alpha}$ where C (t) is cumulative failure rate at times t
γ is the scale parameter
α is the growth parameter

Taking the log of both sides of the equation, the model becomes linear on log-log paper with slope α.

$$\log(C(t)) = \log(\gamma) + \alpha \log(t)$$

The AMSAA model is a statistical model of the same form as the Duane model. It is different in that it is modeling the reliability within a test phase as a function of time accumulated in the test phase.

$N(t) = \lambda t^{\beta}$ N(t) is cumulative failures by time t
λ is the scale parameter
β is the growth parameter

b) Model Assumptions

The Duane model assumes that the cumulative failure rate grows along the curve γt^{α} as a function of development time.

The AMSAA model assumes that the failures are from a nonhomogeneous poisson process (NHPP) with a Weibull intensity function. The intensity function is as follows:

$$\rho(t) = \lambda\beta\, t^{\beta-1}$$

It is further assumed that if no further effort is made to improve the reliability, the failure rate will remain constant (time between failures will follow an exponential distribution). The following assumptions are a result of the NHPP assumption.

- N(0) = 0 (no failure can occur without some test time)
- The probability of a failure at time t is not affected by previous failures
- Simultaneous failures are not possible.

2. Box-Jenkins Time Series Modeling

This modeling procedure is a very good alternative to the AMSAA/Duane model when the quality of data is poor, or the AMSAA model assumptions are in question. This procedure does not require that a specific model form be selected i advance, however, a first difference moving average model is generally accepted. The Box-Jenkins methodology has a built-in theory of forecasting and generally does as well as the AMSAA model in making short term predictions (up to 12 months) of reliability.

a) Model Description

The Box-Jenkins ARIMA (Auto-Regressive-Integrated-Moving-Average) model is a model of reliability against calendar time. The function that defines cumulative failure rate at a given point in calendar time ($\lambda_{c,T}$) is a function of previous failure rates, random errors and parameters. A first difference moving average model is recommended, as follows:

$$\lambda_{c,T} = \theta_0 + \lambda_{c,T-1} + \varepsilon_T - \theta_1 \varepsilon_{T-1}$$

or

$$\lambda_{c,T} - \lambda_{c,T-1} = \theta_0 + \varepsilon_T - \theta_1 \varepsilon_{T-1}.$$

θ_0 is the trend parameter. This parameter may frequently be insignificant and therefore not included. Statistical tests are available (Appendix E) for determining if it is significant.

$\lambda_{c,T} - \lambda_{c,T-1}$ is the first difference part of this first difference moving average and

$\varepsilon_T - \theta_1 \varepsilon_{T-1}$ is a first order weighted (θ_1) moving average of the random error terms.

Occasionally θ_1 will not be significant and the model will be a simple first difference model or sometimes referred to as a random walk.

$$\lambda_{c,T} = \lambda_{c,T-1} + \varepsilon_T$$

b) Model Assumptions

Box-Jenkins Time Series Modeling assumes that the residual effect or variation unexplained by the model is normally distributed with mean 0 and variance σ_a^2. It is further assumed that the reliability is tracked at equally spaced time intervals (e.g. monthly).

3. Endless-Burn-In Model

This model takes into account past burn-in and uses the average age of the population rather than cumulative age. It also states that reliability growth approaches a limiting value (λ_r). The model describes instantaneous failure rate as a function of average test time. This model is intended for solid-state electronic failure modes only. This model was effective in making short term forecast (e.g. 12 months) and the parameters were easier to estimate than the modified Duane model (another 3 parameter model). Honeywell Avionics, St. Louis Park, Mo, has used the model successfully on the Digital Air Data Computer (DADC) for use on the DC-10 aircraft (See Appendix A No. 23).

a) Model Description

The form of the model is as follows:

$$\lambda_i = K\, T_{ave}^{-\alpha} + \lambda_r$$

λ_i is the instantaneous failure rate
λ_r is the limiting failures rate for λ_i
K, α are scale and growth parameters, respectively
T_{ave} is the average fleet time.

b) Model Assumptions

The Endless-Burn-In model assumes that the instantaneous failure rate grows along the curve $K\, T^{-\alpha} + \lambda_r$ as a function of average age or operating time. This assumption implies the following:

- Average age is a strictly increasing function of calendar time. This could be a problem if several new units are introduced into the fleet during the development program. For example, in Figure D-1, during the first 5 years of development 18 units accumulated approximately 56,000 hours for an average of 3111 hours. When the third generation was introduced into the program in the fifth year, the average age dropped to 1168 hours (56,000/48 = 1166) and by the end of the program climbed back to 1510.

- The average age is assumed to be a good estimate of a typical unit on test. This is not the case in the Figure D-1 example, since some units accumulate large amounts of time (CERT units) and others do not accumulate much time (flight test units). In addition to the average age assumption, it is assumed that an adequate amount of test time will be accumulated to estimate or see the effect of the limiting parameters.

An example of a program where these assumptions are met would be the DADC Honeywell data. Here, all units accumulated time at about the same rate and the total program has accumulated over 2 million hours. The model was successfully applied to the electronics failure modes (see the article "Growth Modeling Improves Reliability Predictions," by Bezat, Norquist and Montague, *Annual Reliab. and Maint. Sym., Proc. 1975).*

C. DESIGN THE *IDEAL* GROWTH CURVE FOR TRACKING

In designing the ideal growth curve, the following information is usually required:

- Initial mean time between fai[illegible] (MTBF) (M_I). This is usually estimated from some initial testing.

- The amount of initial testing T_I.

- The final MTBF (M_F). This is usually the reliability goal for the program.

- The cumulative amount of test time allocated for the program T_F.

The model being fit is as follows:

$$M(t) = M_I\left(\frac{t}{T_I}\right)^{\alpha}(1-\alpha)^{-1}$$

The ideal curve has a base value of M_i at time T_I. $M(t)$ increases from T_I to T_F where it reaches the value M.

To approximate in the equation above when α is less than or equal to 0.5, observe the following:

$$\alpha = -\log\left(\frac{T_F}{T_I}\right) - 1 + \left[\left(1 + \log\left(\frac{T_F}{T_I}\right)\right)^2 + 2\log\left(\frac{M_F}{M_I}\right)\right]^{1/2}.$$

For values of $\alpha > 0.5$ (very optimistic programs), use iterative techniques for solving for α.

NOTE: If an additional point on the curve is *known,* α can be calculated directly. Suppose the MTBF at the time t_1 is known to be M_1, then

$$\alpha = \frac{\log(M_F) - \log(M_1)}{\log(T_F) - \log(t_1)}.$$

1. Example of the Calculation of the Growth Parameter

Assume that an initial MTBF of 100 hours has been demonstrated from the first 1000 hours of generation 1 testing, shown previously in Figure D-1. There is a total of 75,650 hours planned for this development program and a 3000 hour MTBF is desired. Note the following formula:

$$\alpha = -\log\left(\frac{75{,}650}{1000}\right) - 1 + \left[\left(1 + \log\left(\frac{75{,}650}{1000}\right)\right)^2 + 2\log\left(\frac{3000}{1000}\right)\right]^{1/2},$$

$$\alpha = 0.605.$$

This indicates that the program is very aggressive. Also note that for $\alpha > 0.5$, an iterative procedure is recommended. In this example, a computer program written in FORTRAN and using IMSL (International Mathematical and Statistical Library) was developed to ensure the accuracy of α. This program shown in Figure D-3 gave an $\alpha = 0.59$. The *ideal growth curve* is also illustrated in Figure D-3. Figure D-4 shows this same curve on log-log scale.

2. Determine the Average MTBF for Each Test Phase

First, compute the expected total failures $N(t_i)$ by the completion of phase i, at time t_i.

$$N(t_i) = \frac{i}{M_I} t_I \left(\frac{t_i}{t_I}\right)^{1-\alpha}$$

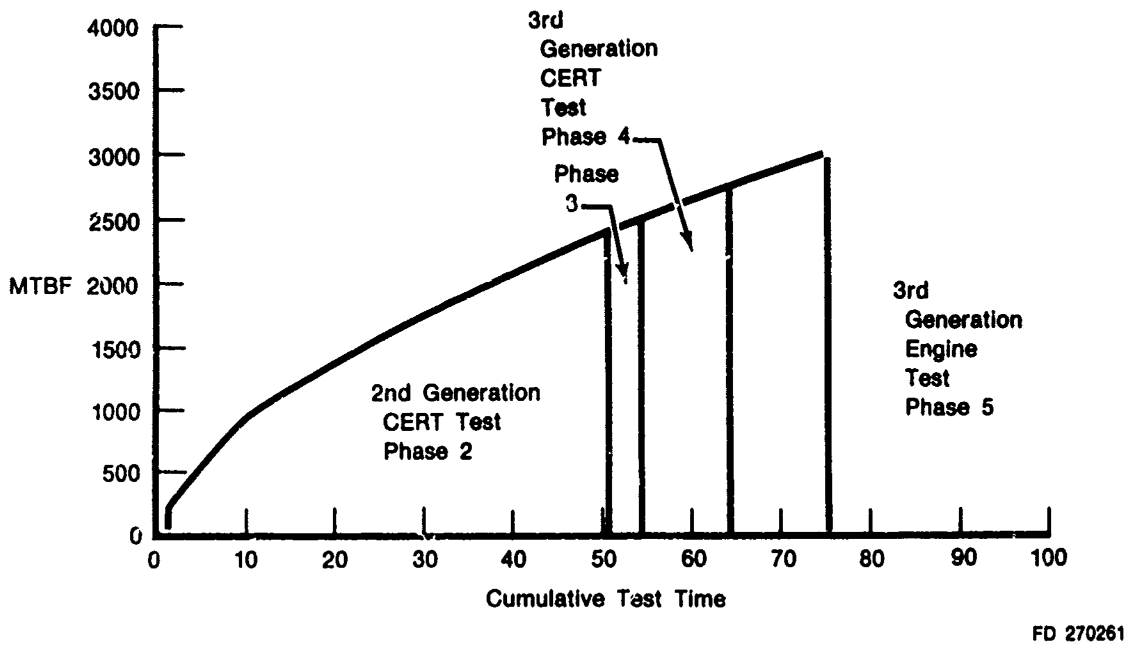

Figure D-3. Ideal Growth Curve for Development Plan

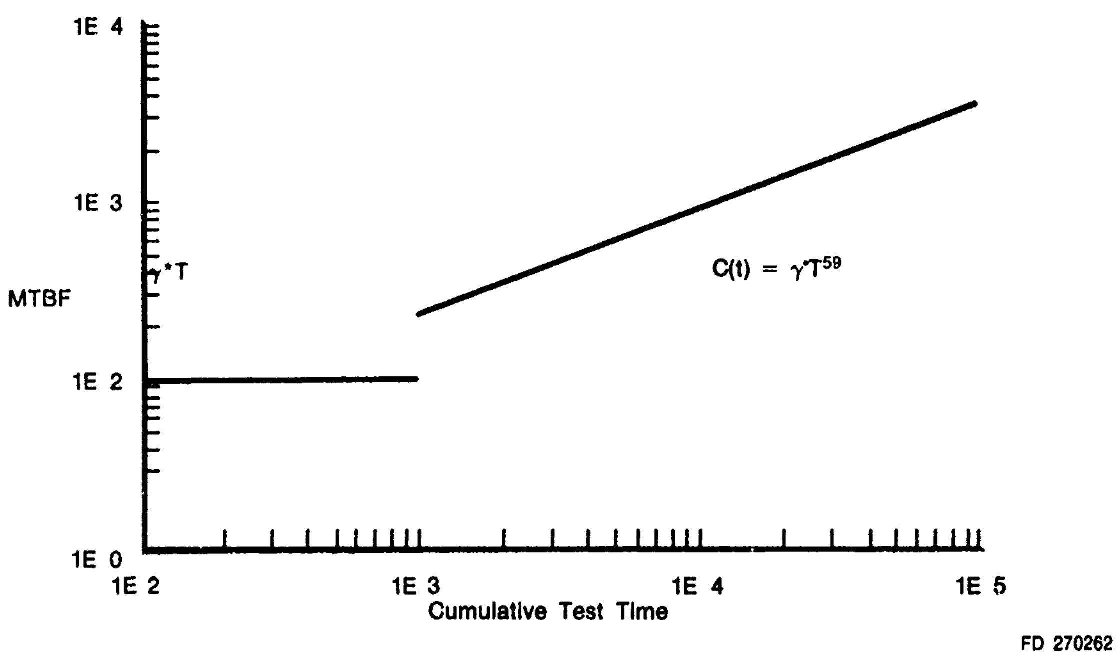

Figure D-4. Ideal Growth Curve for Development Plan

The average MTBF in phase i is as follows:

$$MTBF = \frac{t_i - t_{i-1}}{N(t_i) - N(t_{i-1})}$$

Phase	*t*	*N*	*MTBF*
1	1,000	10.0	100
2	52,000	50.5	1009
3	55,000	51.7	2500
4	65,000	55.4	2702
5	75,650	59.2	2802

Figure D-5 shows the average MTBF sketched over the ideal curve, previously shown in Figure D-3.

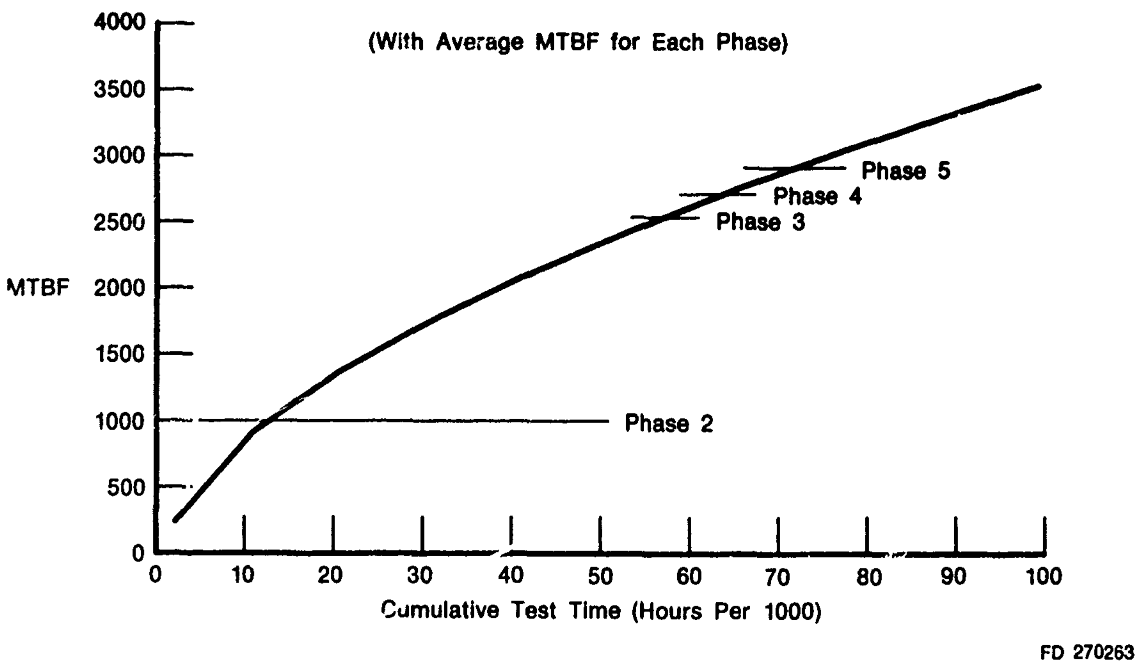

Figure D-5. Ideal Growth Curve for Development Plan (With Average MTBF for Each Phase)

3. Evaluating the Feasibility of the *Ideal* Curve

The *ideal* curve should be compared to past experience to determine if the growth within each phase is reasonable. It should be noted that the *ideal* growth curve does not consider delayed fixes. Therefore the planned curve should consider delayed fixes. Figure D-6 shows that at some point fixes will no longer be implemented until the end of the test.

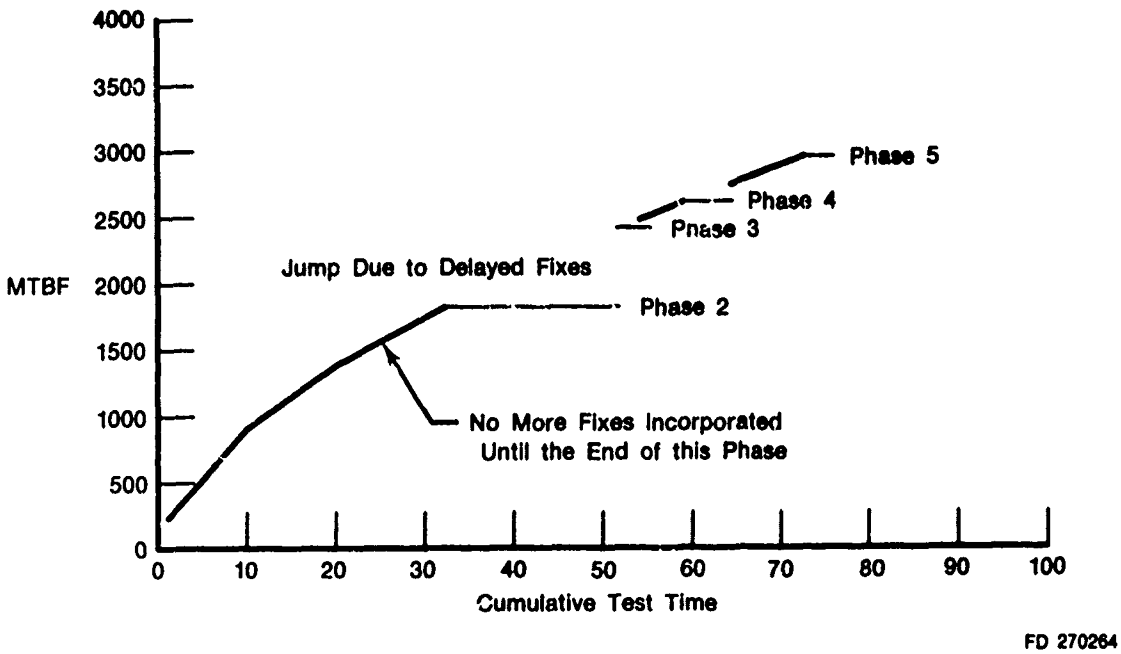

Figure D-6. Ideal Growth Curve for Development Plan

In the example, a growth rate α = 0.59 is considered higher than usual but still obtainable. The rate would be expected to remain constant until the fixes are implemented. A jump would be seen at the start of the next phase as a result of these fixes.

The design is reviewed to determine if the system is considered an evolution of previous controls systems or a system that is considered to be revolutionary. Resources should also be checked to ensure that the goals are obtainable.

D. ANALYSIS OF DATA

Once the program is underway and data has been collected, it will be necessary to analyze the data for purposes of assessment, prediction and control. This analysis involves engineering and statistical methodology with examples using continuous time truncated data and grouped data.

1. Objectives of the Analysis

The primary objectives of the analysis are as follows:

- *Assessment* — An estimate of the current reliability with confidence limit (if possible) should be made to assess the reliability.
- *Prediction* — A forecast of future reliability over some future time period would be helpful for planning purposes.
- *Control* — A comparison of the demonstrated reliability (assessment) to the planned reliability is helpful in determining if the program is on schedule, hence under control.

2. Engineering Analysis of Failure Data

Engineering analysis of failure data is performed to (1) understand the failure modes that exist in a component or system, (2) evaluate design changes to eliminate these failure modes, and (3) apply this information to the reliability growth model that represents the component or system. This analysis is of particular importance where design changes were not incorporated in a particular design phase or where adequate time did not exist to verify the improvements.

a) Preliminary Analysis

Initial analysis of the failure data is conducted to determine completeness and accuracy. It is desirable to have individual unit time at failure, cumulative fleet time at failure, and detailed information on the circumstances surrounding each failure event. This information is generally available during a reliability development program such as CERT, as discussed in Section 1.5. Field data is often less detailed and based on group data (such as monthly). This information, although not as complete as desired, is sufficient to do growth modeling, provided failure investigation information is available.

After failure data has been collected and is considered complete, it must be reviewed to ensure that all reported malfunctions are confirmed to be component failures. In both development testing and field operation, the reported malfunction of a component is often found to be unconfirmed or induced after detailed analysis and testing. The unconfirmed failures are often due to poor troubleshooting, incomplete procedures, or lack of diagnostic equipment. The induced failures are caused by mishandling, misoperation, failures of other hardware, or use beyond established limits. Because these unconfirmed and induced failures are not necessarily a function of the true component reliability, these reported malfunctions should be eliminated from the failure data prior to growth modeling.

b) Failure Mode Assessment

The failure data should be further analyzed to identify failures by mode. Each failure mode should then be reviewed to ensure that sufficient investigation has been completed to understand the reason for failure. At this point the corrective action can be evaluated as to its effectiveness and the expected impact of the changes on reliability growth evaluated. This impact on growth is influenced by the accuracy of the failure mode investigation, the effectiveness of corrective actions, and whether that corrective action is implemented during or after the testing phase.

Table D-1 shows a summary of EEC CERT data to be used as an example.

For each failure during the test, the malfunction cause was identified, each malfunction was investigated, and corrective action formulated, as shown in Table D-1. In this example some corrective actions were implemented during testing and some were implemented as delayed fixes.

The failure mode identified as *Socket Damage* in Table D-1 was investigated and found to be due to inadequate process controls at the time of assembly. Corrective action was formulated and all units retrofitted using the new process controls. No further malfunctions were noted in over 45,000 hours of test, resulting in reliability growth during the CERT test phase.

TABLE D-1. EEC CERT FAILURE SUMMARY

Item Number	*MR Number*	*Unit Number*	*Fault Date*	*UUT Time to Malf.*	*All UUT Time to Malf.*	*Malfunction Cause*	*Failure Class.*	*Corrective Action*
1	E001	6	1/7/81	2.0	4.0	Socket Damage	EMW	Add HS9575 Req'ts
2	E002	3	1/7/81	10.4	20.8	TPT2.Interm.Unk.	Other	Monitor and TBD
3	E003	2	1/13/81	21.9	295.0	Socket Damage	EMW	Add HS9575 Req'ts
4	E004	5	1/13/81	22.5	295.0	Socket Damage	EMW	Add HS9575 Req'ts
5	E005	4	2/17/81	628.0	2459.0	Socket Damage	EMW	Add HS9575 Req'ts
6	E006	3	3/4/81	1070.0	3100.0	Socket Damage	EMW	Add HS9575 Req'ts
7	E008	6	5/12/81	1351.1	7274.0	Incorrect tool use	EMW	Added info + QC test
8	E009	2	6/8/81	1735.7	8996.3	Socket/lead contact	EMD	Process change
9	E012	4	8/18/81	3466.0	15522.1	Socket/lead contact	EMD	Process change
10	E013	3	9/29/81	4651.0	19988.7	Lead abrasion	EMD	In process, TBD
11	E015	2	10/12/81	3395.8	20751.8	Resistor network	PMD	EC14277[illegible]
12	E007	5	10/25/81	3619.8	21840.6	Socket/lead contact	EMD	Process change
13	E020	2	11/[illegible]/81	3822.5	22695.5	Socket/lead contact	EMD	Process change
14	E016-1	3	11/3/81	5176.9	22793.2	Resistor network	PMD	EC142775
15	E016-2	3	11/3/81	5176.9	22793.2	Lead abrasion	EMD	In process, TBD
16	E019	4	1/9/82	4866.0	26872.9	Socket/lead contact	EMD	Process change
17	E021	3	2/24/82	6407.8	29684.0	Socket/lead contact	EMD	Process change
18	E022	2	4/27/82	5706.2	33853.1	Socket/lead contact	EMD	Process change
19	E023	1	5/17/82	5473.5	35342.5	Loose connector	PMD	EC140374
20	E025	3	6/1/82	7672.3	36082.7	Resistor network	Other	None, part damaged
21	E028	1	7/16/82	6586.9	39663.7	Socket/lead contact	EMD	Process change
22	E029	4	8/4/82	5366.0	41662.7	Socket/lead contact	EMD	Process change
23	E030	5	8/9/82	5805.2	42242.1	Socket/lead contact	EMD	Process change
24	E031	3	8/13/82	8867.1	42586.2	Prom, bent lead	EMW	Add HS9575 Req'ts
25	E032	2	8/19/82	7133.4	42998.0	Socket/lead contact	EMD	Process change
26	E037	5	8/20/82	5988.5	43059.4	Res. WW, crimp area	EMD	None/isolated case
27	E033	4	9/4/82	5865.7	45003.0	Socket/lead contact	EMD	Process change
28	E034	1	9/5/82	7044.6	45025.0	Socket/lead contact	PMW	Process change
29	E040	5	10/18/82	6809.4	48507.8	Socket/lead contact	EMD	Process change

The failure mode identified as *Socket/Lead Contact* in Table D-1 was investigated and found to be due to adhesive contamination. Corrective action was identified which changed the adhesive and eliminated this failure mode. Corrective action for this mode however was not implemented until late in the test. Reliability growth, therefore, would not be expected during the test as a result of this delayed fix. Improvement due to this fix is calculated using the techniques described in Section 4.3.6. The effectiveness of each corrective action required by this technique is based upon engineering judgement using all available information. For this failure mode, the cause was well established by detailed failure analysis. The corrective action will eliminate the primary source of contamination found in failed units. Testing of a similar design, however, indicates that contamination from other sources, such as human error in assembly and contact wear, will continue to occur. A corrective action effectiveness for this problem of 75% results in an expected rate of $60/10^6$ hours. Engineering evaluation indicates that this is a reasonable rate for the next phase of testing. Some additional guidance is provided in MIL-HDBK-189 Appendix A for making engineering evaluations for delayed fixes.

Evaluation of field data requires similar techniques as described by this example. Data, however, is usually group data and fixes tend to be delayed.

c) *Continued Engineering Analysis*

Engineering analysis of the data should continue throughout the statistical analysis. This includes review of plots, identification of engineering reasons for any discontinuities observed and estimating the effectiveness of all delayed fixes.

3. Statistical Analysis

Within a test phase, it is assumed that problems are being identified and corrected and, as a result, the reliability of the system is improving. Since the system has been changed, data from the earlier part of the test is not representative of the current systems; however, there is usually a very limited amount of current data. Because of this situation, reliability growth models (such as the AMSAA/Duane model) are employed. These models make use of the combined data in modeling the growth process and making estimates of current and future reliability in the presence of a changing configuration.

a) *Preliminary Analysis of Data*

Before attempting to use the model and make reliability assessments, it is important to review the data for problems such as outliers and missing data. The best approach to such problems is usually to simply look at the data. Plots of cumulative failures vs cumulative time, average failure rate vs cumulative time, and cumulative MTBF vs cumulative time are helpful in identifying problems in the data as well as determining the usefulness of models.

Cumulative Failures vs Cumulative Time

This plot is helpful in assessing the fit of the AMSAA model. If the AMSAA model is appropriate, the data should be linear on log-log paper. Recall that the expected total failures by time T is $N = \lambda T^{\beta}$. Taking the natural logarithm of both sides.

$$\log(N) = \log(\lambda) + \beta \log(T).$$

If there is a distinct break in the data, it may indicate a change in emphasis in the program, significant change in technology level, or an unpredicted change in the testing environment as well as a number of other reasons. It should be reviewed and the data modeled in two parts.

For example, the following data in Table D-2 is from a typical electronic control CERT test which would correspond to the second generation CERT test in the simulated example shown previously in Figures D-1 and -2.

After reviewing the data for obvious problems and reviewing the engineering analysis to ensure that each failure is applicable to the program and not caused as a result of test problems, etc., the cumulative failure vs cumulative time plot is produced (See Figure D-7).

From this plot there appears to be a break at about 20,000 hours. During the first 20,000 hours the growth rate was very high (β = 0.37, or in terms of the Duane curve, α = 0.63). The last 30,000 hours, 20,000 to 50,000, showed very little growth (β = 0.92 or α = 0.078). Figure D-8 shows the first 20,000 hours of data shown in Figure D-7. Figure D-9 shows the last 30,000 hours of data shown in Figure D-7.

TABLE D-2. EEC CERT TEST

Incidents		Unit	Unit Time	Fleet Time
1	E001	6	2.0	4.0
2	E002	3	10.4	20.8
3	E003	2	21.9	295.0
4	E004	5	22.5	295.0
5	E005	4	628.0	2459.0
6	E006	3	1070.0	3100.0
7	E008	6	1351.1	7274.0
3	E009	2	1735.7	8996.3
9	E012	4	3466.0	15522.1
10	E013	3	4651.0	19988.7
11	E015	2	3395.8	20751.8
12	E007	5	3619.8	21840.6
13	E020	2	3822.5	22695.5
14	E016-1	3	5176.9	22793.2
15	E016-2	3	5176.9	22793.2
16	E019	4	4866.0	26872.9
17	E021	3	6407.8	29684.0
18	E022	2	5706.2	33853.1
19	E023	1	5473.5	35542.5
20	E025	3	7672.3	36082.7
21	E028	1	6586.9	39663.7
22	E029	4	5366.0	41662.7
23	E030	5	5805.2	42242.1
24	E031	3	8867.1	42586.2
25	E032	2	7133.4	42998.0
26	E037	5	5988.5	43059.4
27	E033	4	5865.7	45003.0
28	E034	1	7044.6	45025.0
29	E040	5	6809.4	48507.8

Unit	Total Time
1	8,234.5
2	8,396.9
3	9,740.6
4	6,235.3
5	6,930.3
6	10,507.3
	50,044.9

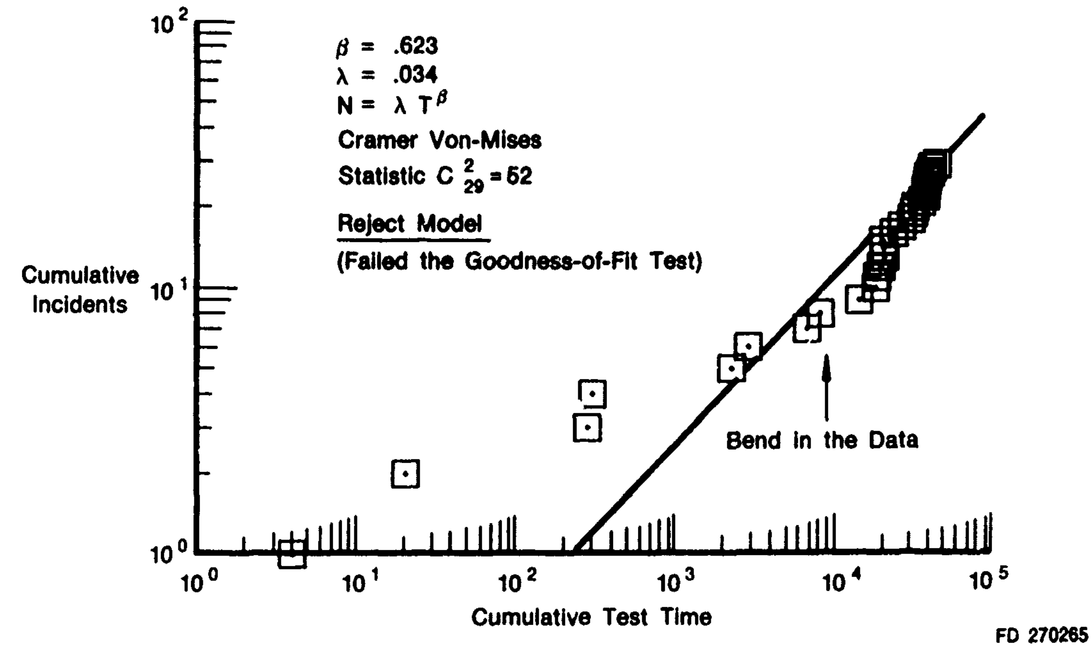

Figure D-7. Reliability Growth of 50K Hour CERT (Cumulative Incidents vs Cumulative Test Time)

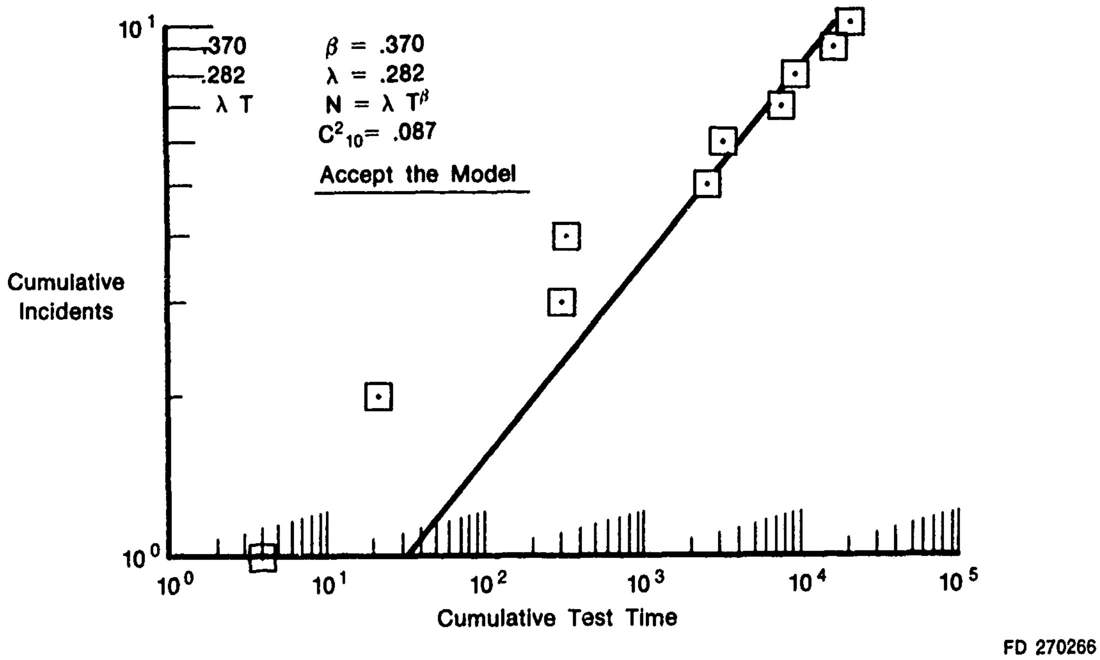

Figure D-8. Reliability Growth of CERT (Cumulative Incidents During First 20K Test Hours)

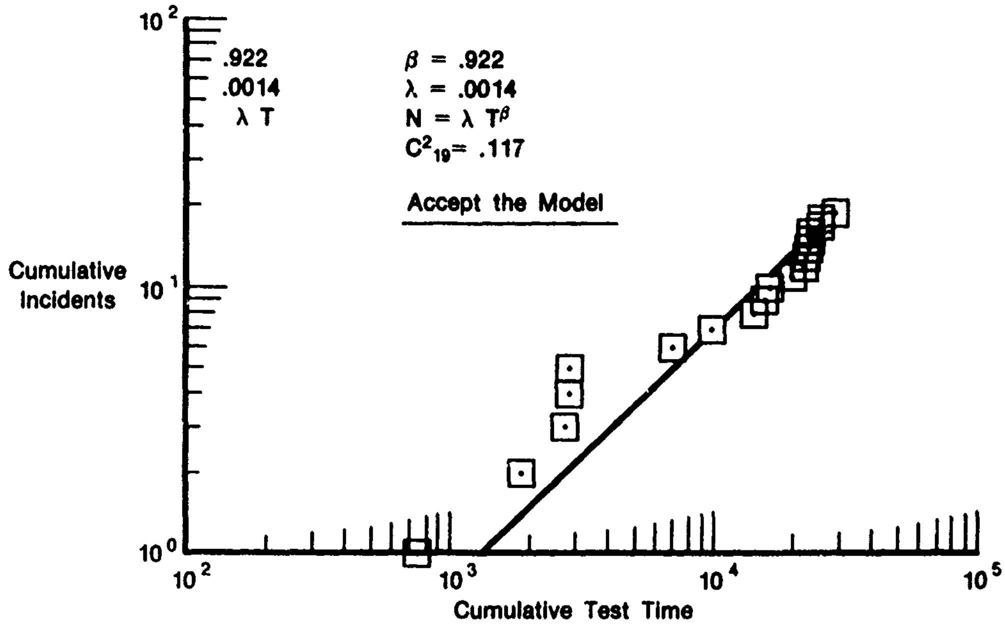

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Figure D-9. Reliability Growth of CERT (Cumulative Incidents During Last 30K Test Hours)

Average Failure Rate Plots

The average failure rate plot shows how the failure rate is changing with time. The average failure rate is computed over each 5,000 hour interval and plotted. Figure D-10 shows all 50,000 hours. Figure D-11 shows the first 20,000 hours and Figure D-12 shows the last 30,000 hours.

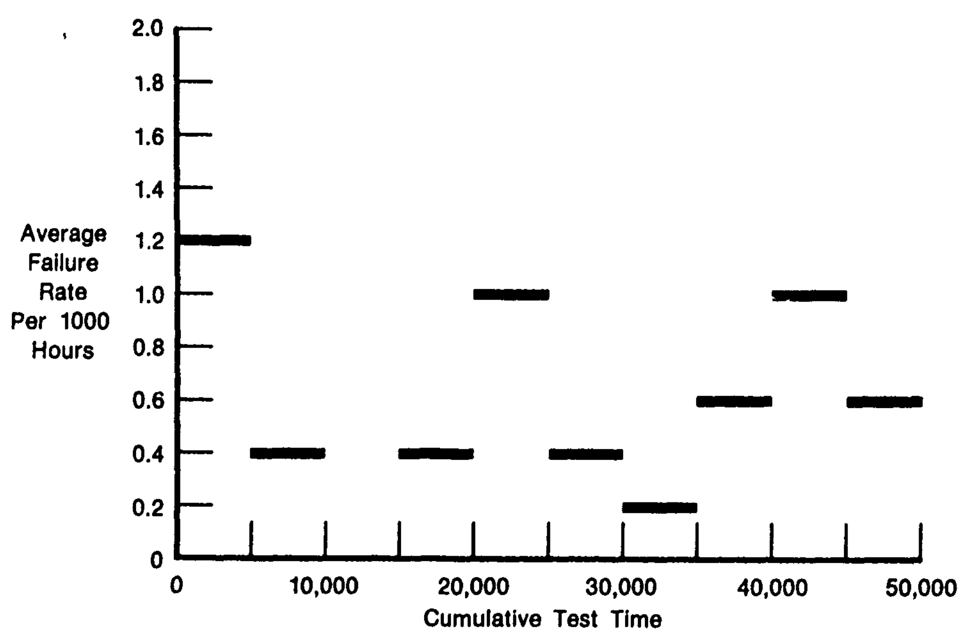

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Figure D-10. CERT Analysis Reliability Growth Analysis Average Failure Rate Plot

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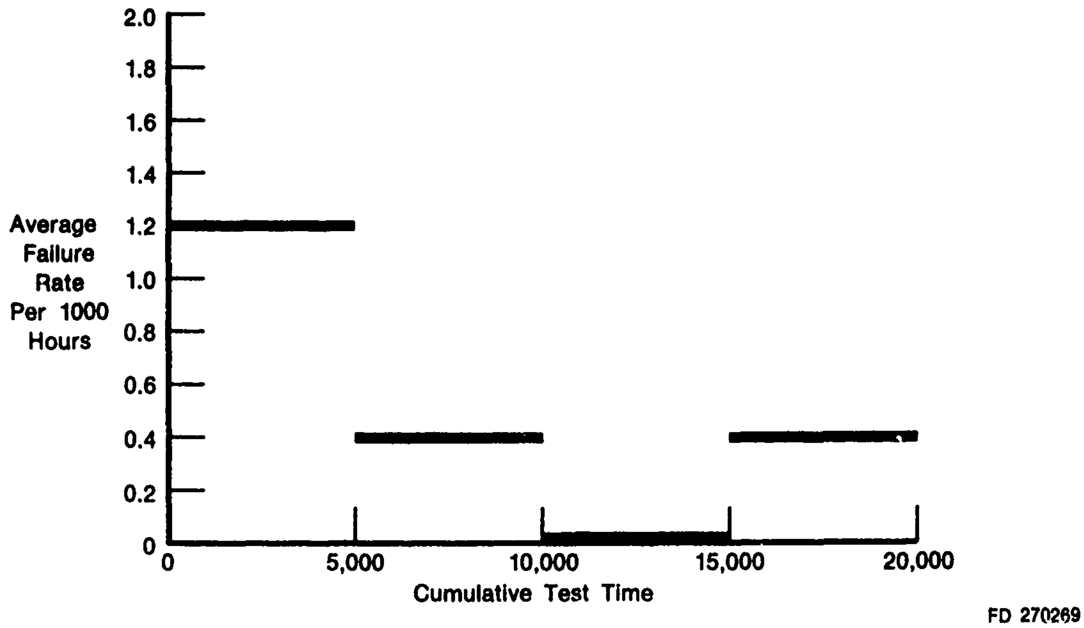

Figure D-11. CERT Analysis Reliability Growth Analysis Average Failure Rate Plot (First 20K Hours)

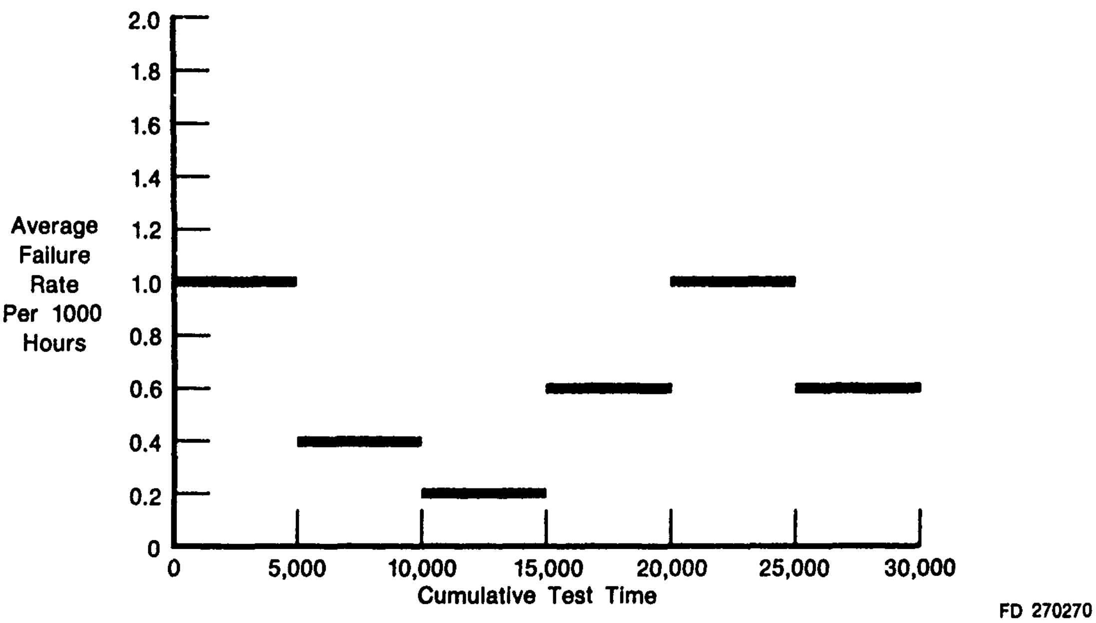

Figure D-12. CERT Analysis Reliability Growth Analysis Average Failure Rate Plot (Last 30K Hours)

Once again, the system appears to have experienced rapid growth over the first 20,000 hours and very little growth thereafter. This plot is also helpful in seeing what the true present configuration failure rate may be.

Cumulative MTBF vs Cumulative Time Plots

This plot illustrates the original learning curve proposed by Duane as shown in Figures D-13, D-14 and D-15, which are transformations of Figures D-7, D-8 and D-9, respectively, and also illustrates the 20,000 hour break.

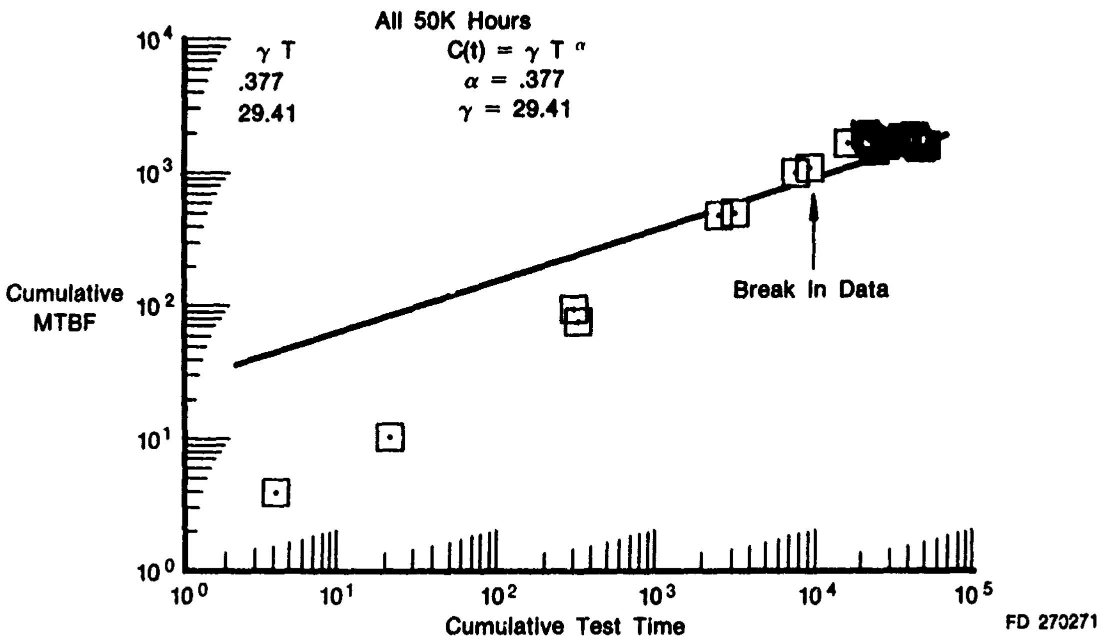

Figure D-13. Reliability Growth of 50K Hour CERT (Cumulative MTBF vs Cumulative Test Time)

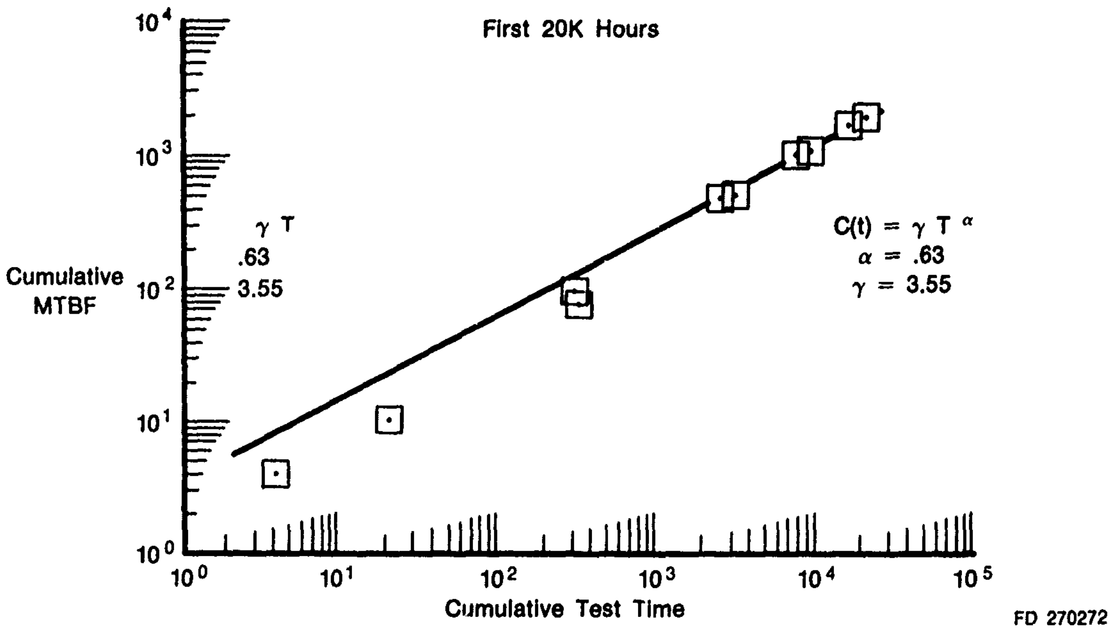

Figure D-14. Reliability Growth of CERT (Cumulative MTBF During First 20K Test Hours)

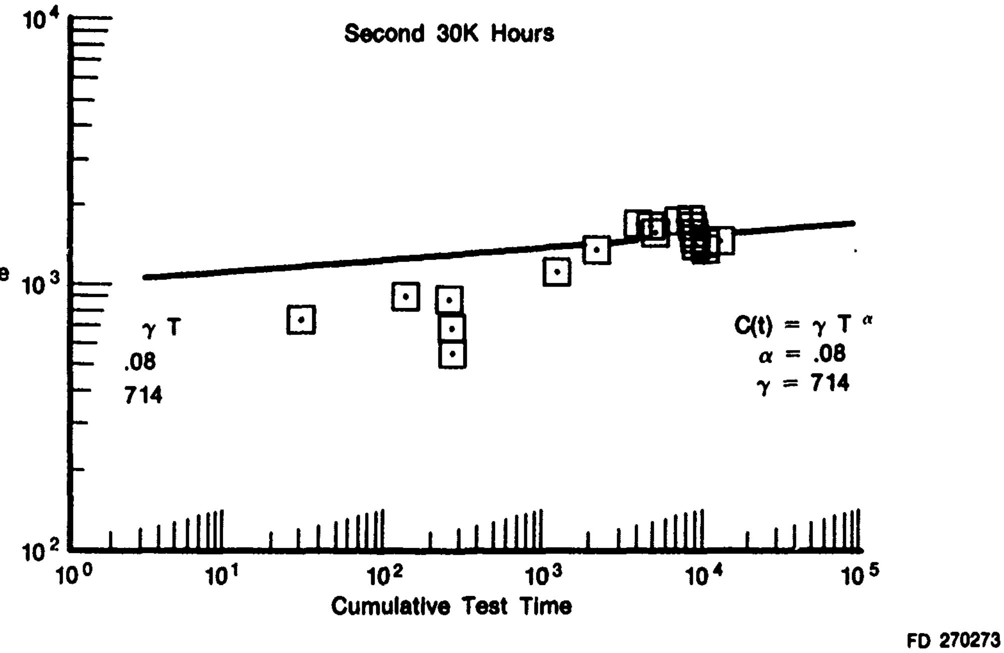

Figure D-15. Reliability Growth of CERT (Cumulative MTBF During Last 30K Test Hours)

Plots Applied to Field Data

While the AMSAA procedure was intended for use within phases of a development program, these are also helpful in analyzing data from field experience where a fleet of controls has been tracked, reviewed and changes made for improving the reliability via retrofit programs. As an example, listed in Table D-3, consider the EEC JFC-90 used on the F100 engine. This data is also illustrated in Figure D-16.

TABLE D-3. EEC FIELD DATA

YEAR ONE CUM TIME	CUM FAIL	YEAR TWO CUM TIME	CUM FAIL	YEAR THREE CUM TIME	CUM FAIL	YEAR FOUR CUM TIME	CUM FAIL	YEAR FIVE CUM TIME	CUM FAIL	YEAR SIX CUM TIME	CUM FAIL	YEAR SEVFN CUM TIME	CUM FAIL
184.0	1.0	10852.8	33.0	65332.8	122.0	196974.1	239.0	413148.1	412.0	721624.6	782.0	1077776.0	1319.0
465.6	1.0	12886.4	37.0	73039.9	127.0	210527.7	253.0	434554.4	435.0	752443.8	833.0	1130756.0	1360.0
731.2	3.0	15256.0	41.0	80839.9	137.0	223817.2	264.0	454584.8	454.0	781784.6	869.0	1166280.0	1400.0
1310.4	4.0	18110.4	50.0	91043.1	148.0	241025.2	276.0	480748.0	494.0	812659.7	900.0	1207144.0	1449.0
1924.8	9.0	21870.4	58.0	100612.7	157.0	257230.0	292.0	507207.2	524.0	846181.3	952.0	1248801.0	1510.0
2662.4	12.0	25348.8	65.0	112110.2	163.0	275647.6	308.0	533784.7	545.0	875002.1	985.0	1287153.0	1555.0
3652.8	14.0	29414.4	76.0	123790.2	179.0	296004.3	316.0	559416.7	581.0	906968.4	1032.0	1331340.0	1601.0
4500.8	17.0	34145.6	84.0	134331.0	189.0	312673.1	336.0	584322.3	613.0	936806.8	1081.0	1376581.0	1653.0
5532 8	22.0	39528.0	95.0	146750.2	203.0	335698.6	354.0	612727.1	655.0	967003.6	1125.0	1414571.0	1700.0
6580.8	23.0	45168.0	106.0	158350.2	209.0	353953.0	373.0	635487.1	695.0	996824.3	1172.0	1451908.0	1738.0
8049.6	25.0	51062.4	108.0	170886.2	220.0	374845.7	386.0	666663.1	722.0	1035277.1	1217.0	1498540.0	1781.0
9102.4	29.0	58145.6	114.0	184127.7	229.0	394063.3	398.0	695765.4	753.0	1065329.0	1261.0	1542906.0	1826.0

Several breaks are apparent here, indicating changes in the system or operating environment.

Average Failure Rate/1000 hours is shown in Figure D-17.
Cumulative MTBF is shown in Figures D-18 and D-19.

The data suggests that the failure rate has been relatively constant over the past 2 years.

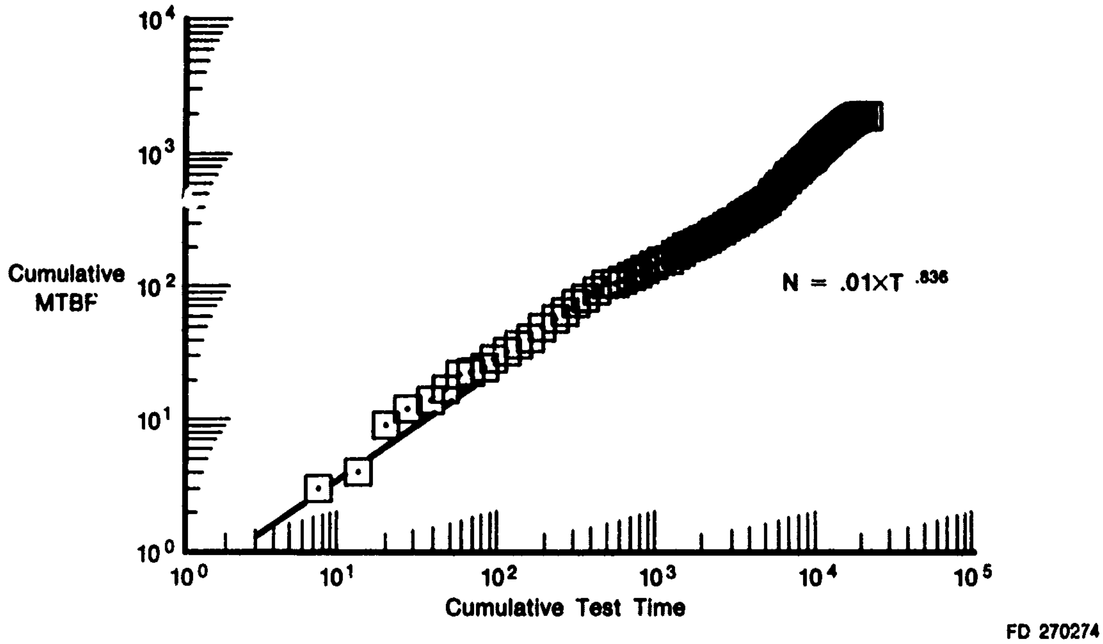

Figure D-16. EEC JFC-90 Field Data (Cumulative Failures)

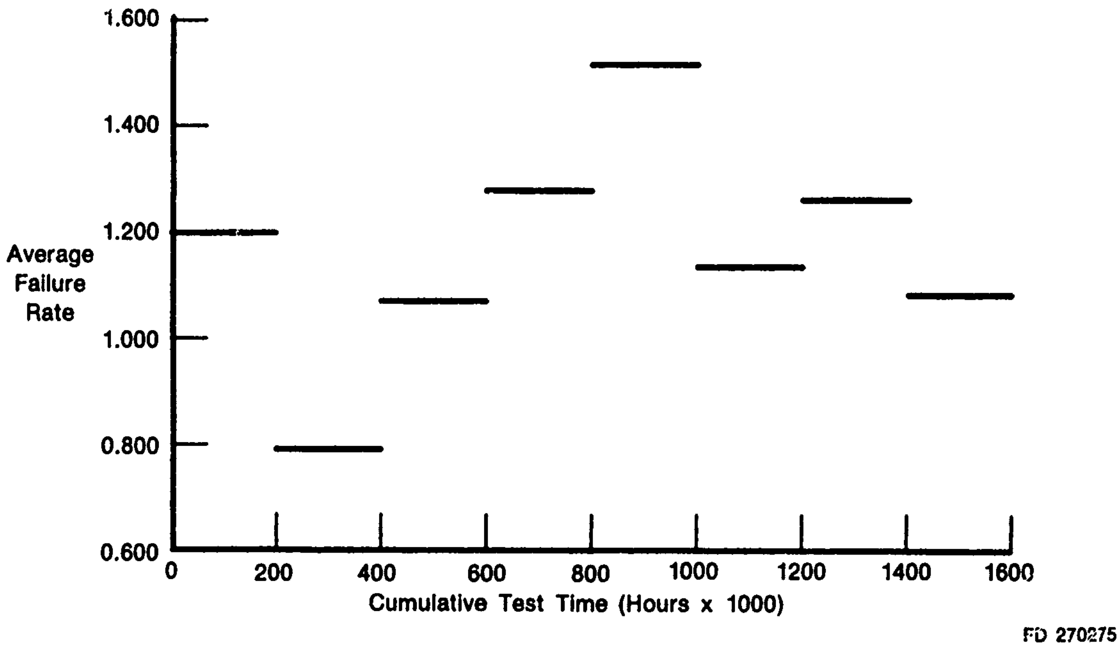

Figure D-17. EEC JFC-90 Field Data (Average Failure Rate)

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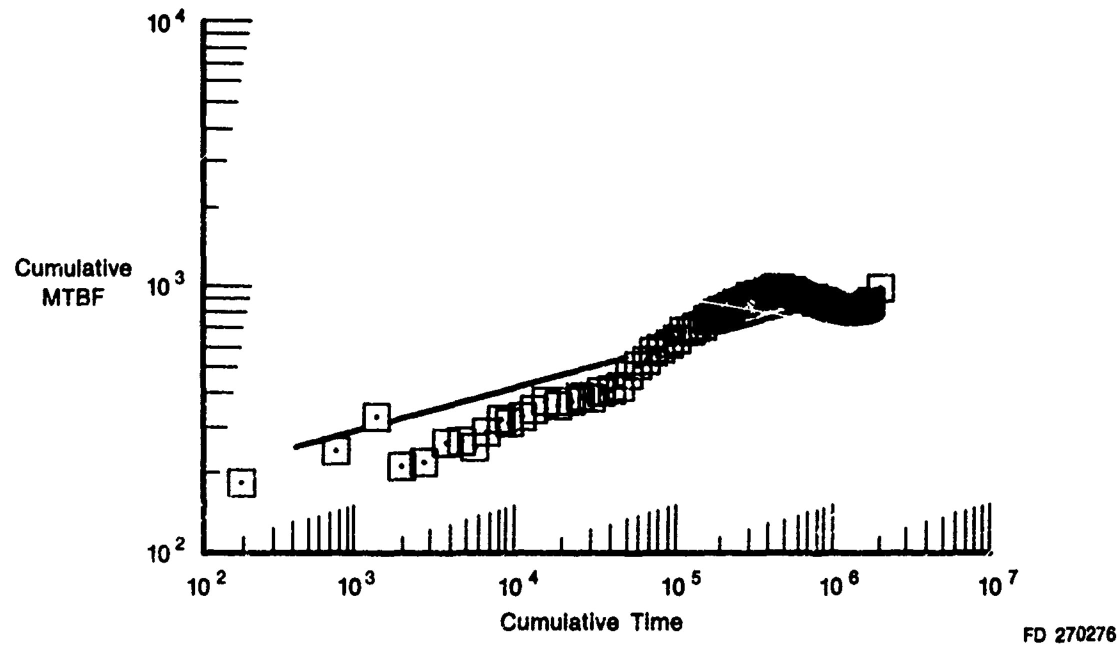

Figure D-18. EEC JFC-90 Field Data (Cumulative MTBF vs Cumulative Time)

Figure D-19. EEC JFC-90 Field Data (Cumulative MTBF vs Calendar Time)

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b) Parameter Estimation

Time-Truncated Data

The EEC CERT test data is from a time-truncated test truncated at 50,044 hours. To establish the AMSAA model parameters, using the maximum likelihood estimations from MIL-HDBK-189,

$$\hat{\beta} = \frac{N}{N \log(T) - \sum_{N-1}^{i=1} \log(t_i)}, \quad \sum_{i=1}^{n} \hat{\lambda} = \frac{N}{T^{\hat{\beta}}}.$$

Where N = the total number of failures, T = the cumulative test time, and t_i is the cumulative test time at the *ith* failure.

The instantaneous failure rate is therefore $\hat{\rho}(T) = \hat{\lambda}\hat{\beta}T^{\hat{\beta}-1}$

and the instantaneous MTBF, $\hat{MTBF} = 1/\hat{\rho}(T)$.

In the first 20,000 hours of CERT test data, $\hat{\beta}$ and $\hat{\lambda}$ would be calculated as follows:

Incident	*Cum Time*	*Log (Cum Time)*
1	4.0	1.386
2	20.8	3.035
3	294.0	5.684
4	315.0	5.753
5	2459.0	7.808
6	3100.0	8.039
7	7274.0	8.892
8	8996.3	9.105
9	15522.1	9.650
10	19988.7	9.903
		69.255

Final Test Total Time 20,000 hours.

$$\hat{\beta} = \frac{10}{10\,(9.903) - 69.189} = \frac{10}{29.846} = 0.335$$

$$\hat{\lambda} = 10 \div (20{,}000)^{0.335} = 0.3623$$

$$\hat{\rho}(20{,}000) = 0.335 \times 0.3623 \times (20{,}000)^{0.335-1} = 1.675 \times 10^{-4}$$

$$\hat{MTBF} = 5970$$

Failure-Truncated Data

Occasionally tests are stopped after a prespecified number of failures have occurred, or when a failure occurs, a decision is made to stop the test until the problem is resolved. These are failure-truncated tests. Maximum likelihood estimates for β and λ are as follows:

$$\hat{\beta} = \frac{N}{(N-1)\log(t_N) - \sum_{N-1}^{i=1} \log(t)}, \quad \sum_{i=1}^{N-1} \hat{\lambda} = \frac{N}{T^{\hat{\beta}}}.$$

If the CERT test were failure truncated at the 10th failure, estimates of β and λ would be as follows:

$$\hat{\beta} = \frac{10}{9(9.903)-(69.189-9.903)} = .335$$

$$\hat{\lambda} = \frac{10}{(19988)^{.335}} = .362$$

which are the same as time-truncated estimates since the last failure occurred so close to the 20,000 hour point.

Grouped Data

If the exact time at which failures occur is not known, the parameters can be estimated from the following equations:

$$\sum_{i=1}^{K} N_i \left[\frac{t_i^{\hat{\beta}} \log(t_i) - t_{i-1}^{\hat{\beta}} \log(t_{i-1})}{t_i^{\hat{\beta}} - t_{i-1}^{\hat{\beta}}} - \log(t_k) \right] = 0.0.$$

Where N_i failures have occurred in the interval t_i, t_{i-1} and K is the number of intervals, shown in the following equation:

$$\hat{\lambda} = \frac{N}{t_k^{\hat{\beta}}}$$

As an example, consider the failure experience on the F100 engine EEC JFC-90 accumulated over a 4 year period of development. Here the failures and time are recorded by month, listed in Table D-4 and shown in Figures D-20, D-21 and D-22. These plots allow the analyst to see the data in Table D-3 pictorially as the AMSAA model sees it. Figure D-20 shows cumulative failures vs cumulative time. As Duane model views it, Figure D-22 shows cumulative MTBF vs cumulative time and Figure D-21 shows the data in failure rate form.

Field Data

It may be appropriate to model field data in the same manner as development, provided the system reliability can be assumed exponential and that a program exists to address new failures as they occur and thus improve system reliability. In most cases, field data will be grouped by month or by quarter.

TABLE D-4. EEC JFC-90 DEVELOPMENT DATA

Time	*Interval*	*Cum Incidents*
0	78	6
78	425	15
425	1,105	19
1,105	2,439	21
2,439	4,776	27
4,776	7,998	31
7,988	11,369	34
11,359	15,263	36
14,263	17,036	37
17,036	20,000	39

Using the above iterative techniques,
β = 0.294 and λ = 2.11

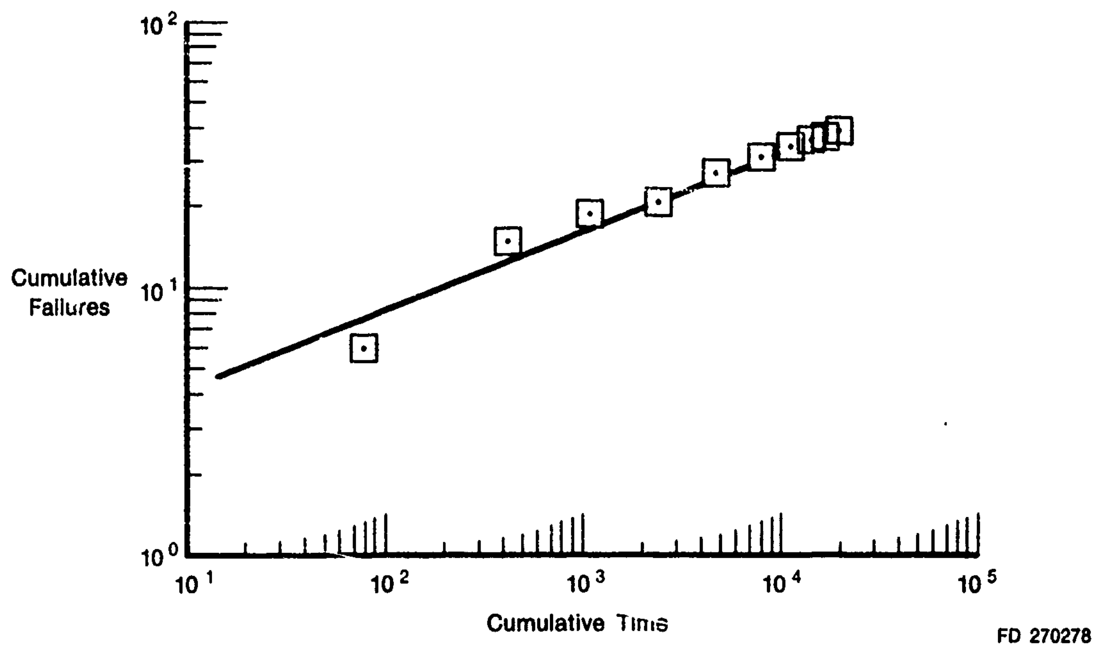

Figure D-20. EEC JFC-90 Development Data (Cumulative Failures)

As an example, consider the F100 engine control field experience over the past 2 years, Figures D-16, 17 and 18.

$\hat{\beta} = 0.836, \qquad \hat{\lambda} = 0.011.$

Regression Estimators/Graphical Methods

Least Squares Regression (LSR) estimates can be obtained by fitting the following Model:

$$\log (N) = \log (\lambda) + \beta \log (T).$$

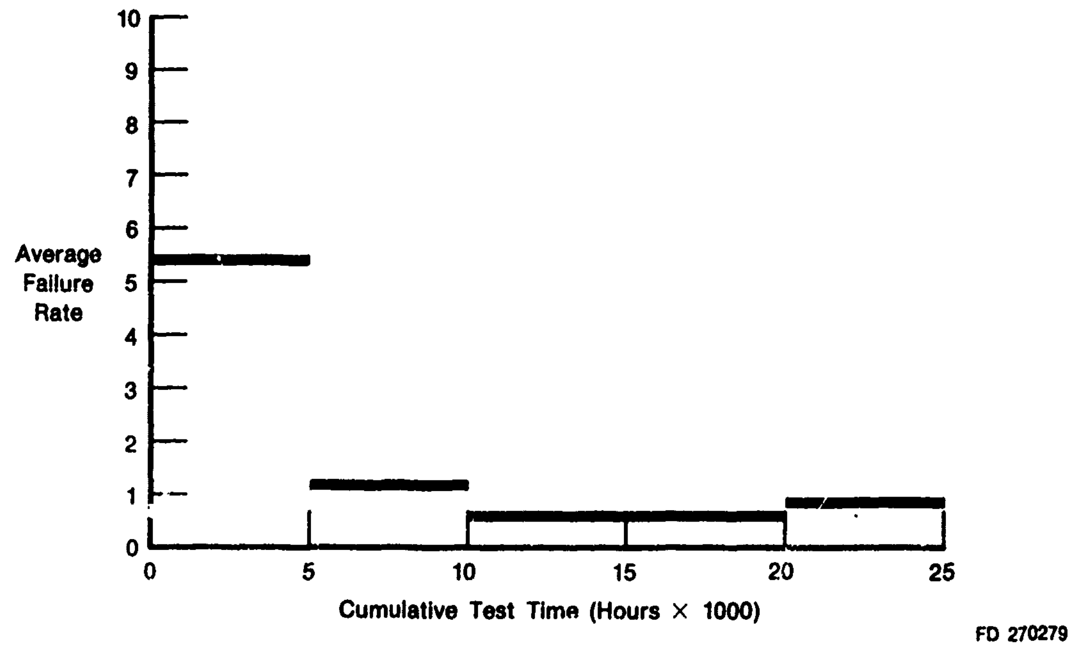

Figure D-21. EEC JFC-90 Development Data (Average Failure Rate)

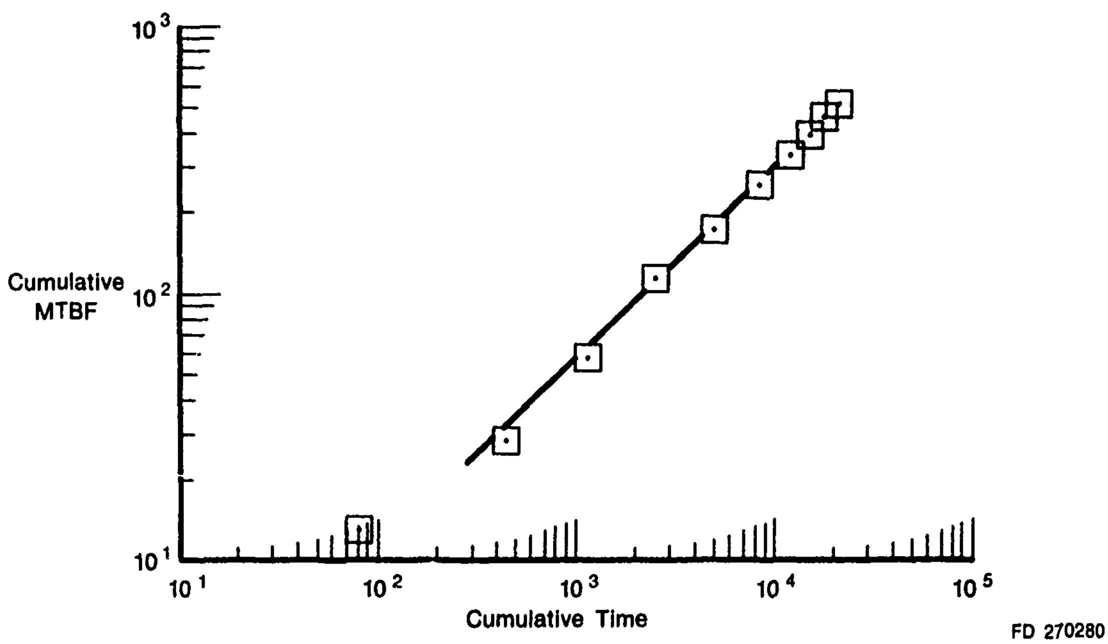

Figure D-22. EEC JFC-90 Development Data (Cumulative MTBF)

LSR estimates are useful in getting the best fit and in getting starting values for the iterative procedure necessary to find maximum likelihood estimators with grouped data. However, Least Squares Estimates are not recommended for use in making inference about the current or future reliability.

Estimates can be obtained graphically as indicated in MIL-HDBK-189, page 133, but these estimates can also be unreliable, and hence are not recommended.

c) Goodness of Fit Test

A statistically valid goodness of fit test should be applied to the data to substantiate that the model is appropriate. If the model and data look comparable graphically and a goodness of fit test indicates the model is appropriate, then inferences drawn from the model are based on a sound foundation.

Continuously Tracked Data

For time-truncated data and failure-truncated data, the Cramer Von Mises statistic (C^2) is used to test the model goodness of fit. The following are needed to calculate this statistic: An unbiased estimate of the growth parameter (β) is used along with the times at each failure (t_i), the total time (T) and the total number of failures (N) to calculate this statistic.

$$\bar{\beta} = \frac{N-1}{N}\hat{\beta} \quad \text{For time-truncated data}$$

$$\bar{\beta} = \frac{N-2}{N}\hat{\beta} \quad \text{For failure-truncated data}$$

$$C_M^2 = \frac{1}{12M} + \sum_{i=1}^{M}\left[\left(\frac{t_i}{T}\right)^{\bar{\beta}} - \frac{2i-1}{2M}\right]^2$$

For time-truncated data, M = N, for failure-truncated data M = N − 1. If the calculated value of C^1_M is less than the table value (at a prespecified significance level, 0.1, 0.05, etc.) then the model is accepted.

For example, with the EEC CERT data, before splitting it into two groups:

$$\hat{\beta} = 0.623,\ N = 29$$

$$\bar{\beta} = \frac{28}{29}\cdot 0.623 = 0.602$$

$$C_{29}^2 = 0.522.$$

The table value listed in Table D-5 is 0.172 for $\alpha = 0.1$; therefore, as was suspected by looking at the graphs in Figure D-7, the model does not fit. After breaking the data at 20,000 hours, models are fit to each new phase as follows:

$\hat{\beta}_1$	= 0.335	$\hat{\beta}_2$	= 0.922
$\bar{\beta}_1$	= 0.302	$\bar{\beta}_2$	= 0.874
$C^2{}_{10}$	= .039	$C^2{}_{19}$	= 0.1167.

Both critical values (0.167 and 0.171) are not less than the calculated values; therefore, modeling the data in two separate parts is appropriate.

TABLE D-5. CRITICAL VALUES FOR CRAMER-VON MISES GOODNESS OF FIT TEST

M/α	*0.20*	*0.15*	*0.10*	*0.05*	*0.01*
2	0.138	0.149	0.162	0.175	0.186
3	0.121	0.135	0.154	0.184	0.23
4	0.121	0.134	0.155	0.191	0.28
5	0.121	0.137	0.160	0.199	0.30
6	0.123	0.139	0.162	0.204	0.31
7	0.124	0.140	0.165	0.208	0.32
8	0.124	0.141	0.165	0.210	0.32
9	0.125	0.142	0.167	0.212	0.32
10	0.125	0.142	0.167	0.212	0.32
11	0.126	0.143	0.169	0.214	0.32
12	0.126	0.144	0.169	0.214	0.32
13	0.126	0.144	0.169	0.214	0.33
14	0.126	0.144	0.169	0.214	0.33
15	0.126	0.144	0.169	0.215	0.33
16	0.127	0.145	0.171	0.216	0.33
17	0.127	0.145	0.171	0.217	0.33
18	0.127	0.146	0.171	0.217	0.33
19	0.127	0.146	0.171	0.217	0.33
20	0.128	0.146	0.172	0.217	0.33
30	0.128	0.146	0.172	0.218	0.33
60	0.128	0.147	0.173	0.220	0.33
100	0.129	0.147	0.173	0.220	0.34

For M > 100 use values for M = 100.

Taken from MIL-HDBK-189

Grouped Data

A Chi-Square goodness of fit test can be used to determine that the growth model adequately represents a set of grouped data. To calculate the Chi-Square statistic, first calculate the expected number of failures in each of the K intervals (e_i), and

$e_i = \hat{\lambda}\,(t_i^{\hat{\beta}} - t_{i-1}^{\hat{\beta}})$, e_i should be greater than 5.

The Chi-Square statistic χ^2, is calculated by comparing the actual number of failures in each interval (N_i) to the expected number of failures (e_i).

$$\chi^2 = \sum_{i=1}^{k} \frac{(N_i - e_i)^2}{e_i} .$$

This statistic (χ^2) has K-2 degrees of freedom. A table value for this statistic can be found by setting the significance level, $\alpha = 0.1, 0.05$, etc., and knowing the degrees of freedom (K-2). If the calculated value of α is less than the table value $\chi^2_{(k-2,\,1-\alpha)}$, accept the model.

Example of F100 electronic engine control development experience is listed in Table D-6.

TABLE D-6. F100 EEC DEVELOPMENT EXPERIENCE

Time	*Interval*	*Cum Incidents*	N_i	e_i	$(N_i - e_i)^2 / e_i$
0	78	6	6	7.6	0.335
78	425	15	9	4.9	3.430
425	1105	19	4	4.1	0.002
1105	2439	21	2	4.3	1.230
2439	4776	27	3	4.6	0.536
4776	7998	31	4	4.1	0.002
7988	11359	34	3	3.3	0.027
11359	14263	36	2	2.2	0.018
14263	17036	37	1	1.9	0.426
17036	20000	39	2	1.8	0.022
				$\chi^2 =$	6.04

Using the above iterative techniques, $\hat{\beta} = 0.294$ and $\hat{\lambda} = 2.11$

The calculated value (6.04) is less than the table value (at $\alpha = 0.9$) (13.4) of Table D-6; therefore, the model is accepted.

Possible Causes for Not Passing the Goodness of Fit Test

If the AMSAA model fails the goodness of fit test, the following courses of action are recommended:

(1) The problem could be bad data. Re-evaluate the preliminary analysis.

(2) Major shifts in the program, reliability or technology level may have gone unnoticed. Recheck the test phases to see if they apply (example with the EEC CERT).

(3) It may be the case that the model just doesn't fit the data. If the data plots a smooth curve on log-log paper, or a snake like curve with no clear/distinct point to break the data, then the AMSAA model may not be appropriate, graphically shown in Figure D-23.

The problem may be that the process is not a nonhomogeneous poisson process (NHPP). Some possible clues that the process is not a NHPP are as follows:

- Can a failure occur before testing begins? NHPP assumes N(0) = 0, i.e., that no failure can occur before time 0. Units with shelf life may violate this assumption.
- Do failures experienced appear to influence the probability of future failures? NHPP assumes that the process has independent increments.
- Have simultaneous failures consistently occurred? NHPP assumes that simultaneous failures are impossible.

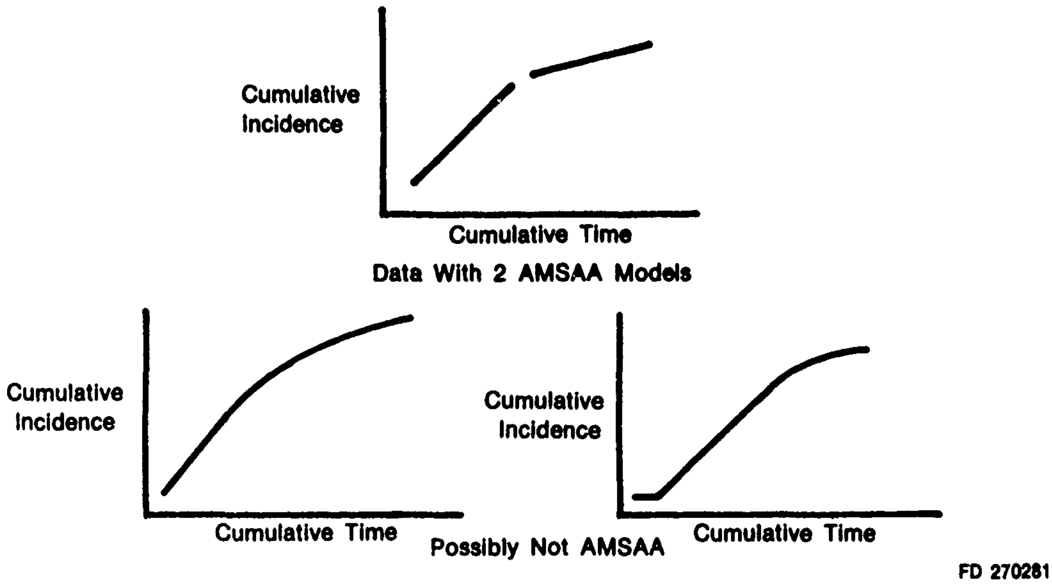

Figure D-23. Examples of Situations Where AMSAA Model May Not Be Applicable

If any of these conditions exist, and a thorough review of the data does not allow for corrective action, then another model should be considered.

Alternate Model

For making short term forecast, when the AMSAA model is not appropriate and sufficient data is available (30+ points), Box-Jenkins Time Series analysis is recommended (See Appendix E).

d) Test for Growth

Statistical tests are available to substantiate that the process is experiencing reliability growth. The type of test depends on the type of available data.

Testing for Growth With Time- or Failure-Truncated Data

When time-or failure-truncated data is available, place interval estimates on β. If these estimates cover 1., growth cannot be substantiated statistically, as illustrated in Figure D-24.

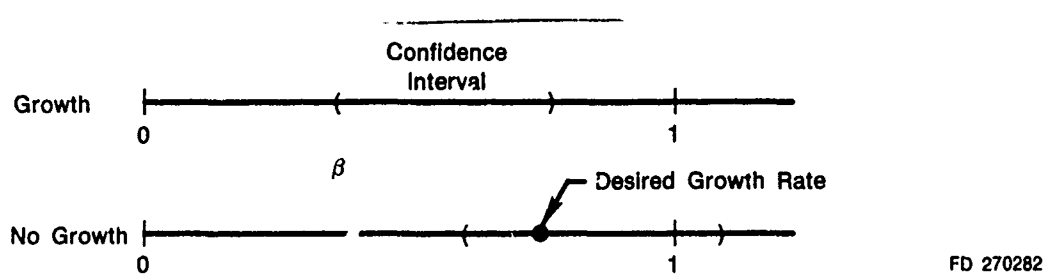

Figure D-24. Geometric Illustration of Growth or No-Growth Situations

If the desired growth rate for this test phase is in the confidence interval, the program is preceding as planned.

Interval estimates for β can be obtained for time-truncated data as follows:

- Find the tabled value of Chi-Square with 2N degrees of freedom. $\chi^2_{2N,\alpha/2}$ and $\chi^2_{2N,1-\alpha/2}$ (see Table D-7).

  $$\text{Lower Bound} = \chi^2_{2N,\alpha/2} \cdot \hat{\beta}/2N$$

  $$\text{Upper Bound} = \chi^2_{2N,1-\alpha/2} \cdot \hat{\beta}/2N$$

- For failure-truncated data.

  $$\text{Lower Bound} = \chi^2_{2N-2,\alpha/2} \cdot \hat{\beta}/2N$$

  $$\text{Upper Bound} = \chi^2_{2N-2,1-\alpha/2} \cdot \hat{\beta}/2N$$

- Consider the example of the first 20,000 hours of CERT data. N = 10, $\hat{\beta}$ = 0.336 at the 90% confidence level (α = 0.1).

  $$\chi^2_{20,0.05} = 10.9$$

  $$\chi^2_{20,0.95} = 31.4$$

$$\text{Lower Bound} = 10.9 * \left(\frac{0.336}{20}\right) = 0.18$$

$$\text{Upper Bound} = 31.4 * \left(\frac{0.336}{20}\right) = 0.53$$

Therefore, based on the AMSAA model, growth is occurring with 90% confidence. The true growth parameter is between 0.18 and 0.53 with 90% confidence.

Testing for Growth with Grouped Data

There is a Chi-Square test for trend that can be applied when only grouped data is available. The Chi-Square statistic (χ^2) is computed and compared to the table value, $\chi^2_{K-1,1-\alpha}$, where 1-α is the confidence level (90%). If the calculated value is greater than the table value, and the calculated value for β, ($\hat{\beta}$) is less than 1.0, growth can be substantiated.

To calculate the Chi-Square value, use the following formula:

$$\chi^2 = \sum_{i=1}^{K} \frac{(N_i - NP_i)^2}{NP_i}$$

Where K is the number of intervals (each interval should be large enough so that $N P_i > 5$), N is the number of failures in interval i, $P_i = t_i / T$, t_i is the time at the upper limit of interval i, $T = \Sigma t_i$, $N = \Sigma N_i$.

TABLE D-7. PERCENTAGE POINTS, CHI-SQUARE DISTRIBUTION

$$F(X^2) = \int_0^{X^2} \frac{1}{2^{n/2}\,\Gamma\left(\frac{n}{2}\right)} X^{n-2/2} e^{-X/2} dx$$

n \ F	.005	.010	.025	.050	.100	.250	.500	.750	.900	.950	.975	.990	.995
1	.0000393	.000157	.000982	.00393	.0158	.102	.455	1.32	2.71	3.84	5.02	6.63	7.88
2	.0100	.0201	.0506	.103	.211	.575	1.39	2.77	4.61	5.99	7.38	9.21	10.6
3	.0717	.115	.216	.352	.584	1.21	2.37	4.11	6.25	7.81	9.35	11.3	12.8
4	.207	.297	.484	.711	1.06	1.92	3.36	5.39	7.78	9.49	11.1	13.3	14.9
5	.412	.554	.831	1.15	1.61	2.67	4.35	6.63	9.24	11.1	12.8	15.1	16.7
6	.676	.872	1.24	1.64	2.20	3.45	5.35	7.84	10.6	12.6	14.4	16.8	18.5
7	.989	1.24	1.69	2.17	2.83	4.25	6.35	9.04	12.0	14.1	16.0	18.5	20.3
8	1.34	1.65	2.18	2.73	3.49	5.07	7.34	10.2	13.4	15.5	17.5	20.1	22.0
9	1.73	2.09	2.70	3.33	4.17	5.90	8.34	11.4	14.7	16.9	19.0	21.7	23.6
10	2.16	2.56	3.25	3.94	4.87	6.74	9.34	12.5	16.0	18.3	20.5	23.2	25.2
11	2.60	3.05	3.82	4.57	5.58	7.58	10.3	13.7	17.3	19.7	21.9	24.7	26.8
12	3.07	3.57	4.40	5.23	6.30	8.44	11.3	14.8	18.5	21.0	23.3	26.2	28.3
13	3.57	4.11	5.01	5.89	7.04	9.30	12.3	16.0	19.8	22.4	24.7	27.7	29.8
14	4.07	4.66	5.63	6.57	7.79	10.2	13.3	17.1	21.1	23.7	26.1	29.1	31.3
15	4.60	5.23	6.26	7.26	8.55	11.0	.4.3	18.2	22.3	25.0	27.5	30.6	32.8
16	5.14	5.81	6.91	7.96	9.31	11.9	15.3	19.4	23.5	26.3	28.8	32.0	34.3
17	5.70	6.41	7.56	8.67	10.1	12.8	16.3	20.5	24.8	27.6	30.2	33.4	35.7
18	6.26	7.01	8.23	9.39	10.9	13.7	17.3	21.6	26.0	28.9	31.5	34.8	37.2
19	6.84	7.63	8.91	10.1	11.7	14.6	18.3	22.7	27.2	30.1	32.9	36.2	38.6
20	7.43	8.26	9.59	10.9	12.4	15.5	19.3	23.8	28.4	31.4	34.2	37.6	40.0
21	8.03	8.90	10.3	11.6	13.2	16.3	20.3	24.9	29.6	32.7	35.5	38.9	41.4
22	8.64	9.54	11.0	12.3	14.0	17.2	21.3	26.0	30.8	33.9	36.8	40.3	42.8
23	9.26	10.2	11.7	13.1	14.8	18.1	22.3	27.1	32.0	35.2	38.1	41.6	44.2
24	9.89	10.9	12.4	13.8	15.7	19.0	23.3	28.2	33.2	36.4	39.4	43.0	45.6
25	10.5	11.5	13.1	14.6	16.5	19.9	24.3	29.3	34.4	37.7	40.6	44.3	46.9
26	11.2	12.2	13.8	15.4	17.3	20.8	25.3	30.4	35.6	38.9	41.9	45.6	48.3
27	11.8	12.9	14.6	16.2	18.1	21.7	26.3	31.5	36.7	40.1	43.2	47.0	49.6
28	12.5	13.6	15.3	16.9	18.9	22.7	27.3	32.6	37.9	41.3	44.5	48.3	51.0
29	13.1	14.3	16.0	17.7	19.8	23.6	28.3	33.7	39.1	42.6	45.7	49.6	52.3
30	13.8	15.0	16.8	18.5	20.6	24.5	29.3	34.8	40.3	43.8	47.0	50.9	53.7

*CRC Handbook of Tables for Probability and Statistics, 2nd Edition, CRC Press.

For example, consider the F100 EEC field experience, shown in Table D-8.

TABLE D-8. EEC JFC-90 FIELD DATA

Time	N_i	$\frac{(N_i - NP_i)^2}{NP_i}$
0-200	239	0.11
200-400	159	23.97
400-600	215	1.52
600-800	256	2.10
800-1000	303	20.43
1000-1200	228	0.15
1200-1400	253	1.56
1400-1600	218	1.08
	1871	$51.76 = \chi^2$

$N = \Sigma N_i = 1871$
$p = t/T = 200/1600 - 0.125$

$\chi^2_{7,.9} = 12.0$, so reject that the data has a constant failure rate. However, two intervals 1) 200-400 and 2) 800-1000 appear to be extremes. Before growth is concluded, these two intervals should be thoroughly researched.

e) *Estimates of Reliability*

Results of the statistical analysis should include an assessment of the current reliability and a projection of future reliability.

Estimate of Current Reliability

It is important to note that the observed reliability at a particular point in time (T) is only an estimate of the true reliability. For example, in the EEC CERT test, at 20,000 hours, it is possible to have obtained a different set of failures had a different set of hardware been tested.

To estimate the current instantaneous MTBF, (M(t))

$$\hat{M}(t) = \frac{1}{\hat{\lambda}} \cdot \frac{1}{\hat{\beta}} T^{1-\hat{\beta}}$$

In the CERT test example,

$$\hat{M}(t) = \frac{1}{0.359} \cdot \frac{1}{0.335} (20{,}000)^{1-0.335} = 6026.$$

To estimate the instantaneous failure rate,

$$\hat{\rho}(t) = \hat{\lambda}\hat{\beta}\, T^{\hat{\beta}-1} = \frac{1}{\hat{M}(t)} .$$

An interval estimate of the instantaneous MTBF (M(t)) can be obtained for time- and failure-truncated data as follows:

- Time-truncated data, look up values of L and U in Table D-9 for N failures and confidence γ.

  $$\text{Lower Bound} = \hat{M}(t) \cdot L$$

  $$\text{Upper Bound} = \hat{M}(t) \cdot U$$

- For failure-truncated data, look up values of L and U in Table D-10 for N failures and confidence γ.

To illustrate the use of confidence bounds on the instantaneous MTBF, consider the EEC CERT data, M(t) = 6026 N = 10, at the 90% confidence level, L = 0.476 and U = 2.575. Therefore, the true reliability would be expected to be between 2868 hours and 15,517 hours with 90% confidence as follows:

0.476*6026 = 2,868
2.575*6026 = 15,517.

This reliability demonstrated over the next 5000 hours was considerably less than the lower bound (5000/5 = 1000 hr MTBF). Engineering analysis suggests that this was a result of continued acceleration of unfixed failure modes from the first 20,000 hour interval.

Estimates of Future Reliability

To compute an estimate of the reliability at a future time T, evaluate M(t) at T as follows:

$$\hat{M}(t) = \frac{1}{\hat{\lambda}} \cdot \frac{1}{\hat{\beta}} \cdot T^{1-\hat{\beta}}.$$

This prediction assumes that the reliability growth continues as in the past. Historically, this estimate has not been reliable beyond 1 to 2 years in advance.

For long term predictions, or predicting beyond two years), use the planned curve to forecast reliability. The expected cumulative failure (C(t)) rate is given by the function:

$$C(t) = \gamma T^{\alpha}$$

$$M(t) = \alpha\gamma T^{\alpha-1}.$$

The assumption made in this projection is that the program obtains its planned growth.

To project reliability within the third phase of the EEC 103-2 development program, assume the growth rate continues at the same rate as the last phase of CERT.

$$\hat{\beta} = 0.922, \hat{\lambda} = \frac{19}{(30{,}000)^{0.922}} = 1.415\times10^{-3}$$

$$\hat{M}(T) = \frac{1}{\hat{\lambda}} \frac{1}{\hat{\beta}} T^{1-\hat{\beta}} = \frac{1}{0.922 \cdot 1.415 \cdot 10^{-3}} (33{,}000)^{1-0.922} = 1225.0$$

TABLE D-9. CONFIDENCE INTERVALS FOR MTBF FROM TIME TERMINATED TEST*

N \ γ	.80 L	.80 U	.90 L	.90 U	.95 L	.95 U	.98 L	.98 U
2	.261	18.66	.200	38.66	.159	78.66	.124	198.7
3	.333	6.326	.263	9.736	.217	14.55	.174	24.10
4	.385	4.243	.312	5.947	.262	8.093	.215	11.81
5	.426	3.386	.352	4.517	.300	5.862	.250	8.043
6	.459	2.915	.385	3.764	.331	4.738	.280	6.254
7	.487	2.616	.412	3.298	.358	4.061	.305	5.216
8	.511	2.407	.436	2.981	.382	3.609	.328	4.539
9	.531	2.254	.457	2.750	.403	3.285	.349	4.064
10	.549	2.136	.476	2.575	.421	3.042	.367	3.712
11	.565	2.041	.492	2.436	.438	2.852	.384	3.441
12	.579	1.965	.507	2.324	.453	2.699	.399	3.226
13	.592	1.901	.521	2.232	.467	2.574	.413	3.050
14	.604	1.846	.533	2.153	.480	2.469	.426	2.904
15	.614	1.800	.545	2.087	.492	2.379	.438	2.781
16	.624	1.759	.556	2.029	.503	2.302	.449	2.675
17	.633	1.723	.565	1.978	.513	2.235	.460	2.584
18	.642	1.692	.575	1.933	.523	2.176	.470	2.503
19	.650	1.663	.583	1.893	.532	2.123	.479	2.432
20	.657	1.638	.591	1.858	.540	2.076	.488	2.369
21	.664	1.615	.599	1.825	.548	2.034	.496	2.313
22	.670	1.594	.606	1.796	.556	1.996	.504	2.261
23	.676	1.574	.613	1.769	.563	1.961	.511	2.215
24	.682	1.557	.619	1.745	.570	1.929	.518	2.173
25	.687	1.540	.625	1.722	.576	1.900	.525	2.134
26	.692	1.525	.631	1.701	.582	1.873	.531	2.098
27	.697	1.511	.636	1.682	.588	1.848	537	2.068
28	.702	1.498	.641	1.664	.594	1.825	.543	2.035
29	.706	1.486	.646	1.647	.599	1.803	.549	2.006
30	.711	1.475	.651	1.631	.604	1.783	.554	1.980
35	.729	1.427	.672	1.565	.627	1.699	.579	1.870
40	.745	1.390	.690	1.515	.646	1.635	.599	1.788
45	.758	1.361	.705	1.476	.662	1.585	.617	1.723
50	.769	1.337	.718	1.443	.676	1.544	.632	1.671
60	.787	1.300	.739	1.393	.700	1.481	.657	1.591
70	.801	1.272	.756	1.356	.718	1.435	.678	1.533
80	.813	1.251	.769	1.328	.734	1.399	.695	1.488
100	.831	1.219	.791	1.286	.758	1.347	.722	1.423

For N > 100,

$$L = (1 + Z_{0.5+\gamma/2}/\sqrt{2N})^{-2} \quad U = (1 + Z_{0.5+\gamma/2}/\sqrt{2N})^{-2}$$

in which $Z_{0.5+\gamma/2}$ is the $(0.5 + \gamma/2)$-th percentile of the standard normal distribution.

*Taken from MIL-HDBK-189.

TABLE D-10. CONFIDENCE INTERVALS FOR MTBF FROM FAILURE TERMINATED TEST*

γ → / N ↓	.80 L	.80 U	.90 L	.90 U	.95 L	.95 U	.98 L	.98 U
2	.8065	33.76	.5552	72.67	.4099	151.5	.2944	389.9
3	.6840	8.927	.5137	14.24	.4054	21.96	.3119	37.60
4	.6601	5.328	.5174	7.651	.4225	10.65	.3368	15.96
5	.6568	4.000	.5290	5.424	.4415	7.147	.3603	9.935
6	.6600	3.321	.5421	4.339	.4595	5.521	.3815	7.388
7	.6656	2.910	.5548	3.702	.4760	4.595	.4003	5.963
8	.6720	2.634	.5668	3.284	.4910	4.002	.4173	5.074
9	.6787	2.436	.5780	2.989	.5046	3.589	.4327	4.469
10	.6852	2.287	.5883	2.770	.5171	3.286	.4467	4.032
11	.6915	2.170	.5979	2.600	.5285	3.054	.4595	3.702
12	.6975	2.076	.6067	2.464	.5391	2.870	.4712	3.443
13	.7033	1.998	.6150	2.353	.5488	2.721	.4821	3.235
14	.7087	1.933	.6227	2.260	.5579	2.597	.4923	3.064
15	.7139	1.877	.6299	2.182	.5664	2.493	.5017	2.921
16	.7188	1.829	.6367	2.144	.5743	2.404	.5106	2.800
17	.7234	1.788	.6431	2.056	.5818	2.327	.5189	2.695
18	.7278	1.751	.6491	2.004	.5888	2.259	.5267	2.604
19	.7320	1.718	.6547	1.959	.5954	2.200	.5341	2.524
20	.7360	1.688	.6601	1.918	.6016	2.147	.5411	2.453
21	.7398	1.662	.6652	1.881	.6076	2.099	.5478	2.390
22	.7434	1.638	.6701	1.848	.6132	2.056	.5541	2.333
23	.7469	1.616	.6747	1.818	.6186	2.017	.5601	2.281
24	.7502	1.596	.6791	1.790	.6237	1.982	.5659	2.235
25	.7534	1.578	.6833	1.765	.6286	1.949	.5714	2.192
26	.7565	1.561	.6873	1.742	.6333	1.919	.5766	2.153
27	.7594	1.545	.6912	1.720	.6378	1.892	.5817	2.116
28	.7622	1.530	.6949	1.700	.6421	1.866	.5865	2.083
29	.7649	1.516	.6985	1.682	.6462	1.842	.5912	2.052
30	.7676	1.504	.7019	1.664	.6502	1.820	.5957	2.023
35	.7794	1.450	.7173	1.592	.6681	1.729	.6158	1.905
40	.7894	1.410	.7303	1.538	.6832	1.660	.6328	1.816
45	.7981	1.378	.7415	1.495	.6962	1.606	.6476	1.747
50	.8057	1.352	.7513	1.460	.7076	1.562	.6605	1.692
60	.8184	1.312	.7678	1.407	.7267	1.496	.6823	1.607
70	.8288	1.282	.7811	1.367	.7423	1.447	.7000	1.546
80	.8375	1.259	.7922	1.337	.7553	1.409	.7148	1.499
100	.8514	1.225	.8100	1.293	.7759	1.355	.7384	1.431

For N > 100,

$$L = [1 + \sqrt{2/N}\, Z_{0.5} + \gamma/2]^{-2} \quad U = [1 + \sqrt{2/N}\, Z_{0.5} + \gamma/2]_{-2}$$

in which $Z_{0.5+\gamma/2}$ is the $(0.5 + \gamma/2)$-th percentile of the standard normal distribution.

*Taken from MIL-HDBK-189

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f) Estimates of Jumps in Reliability

Occasionally, it is necessary to estimate the amount of improvement in reliability as a result of several delayed fixes. This situation usually occurs at the end of a test phase, but can occur elsewhere. The following assumptions are necessary to estimate the jumps:

- The system reliability at time T, (R(T)), is a result of two categories of failure modes: A ~ modes which are not fixed and B ~ modes for which delayed fixes are available.

- Each failure mode in the system has a constant failure rate, λ_i.

  $\lambda_A = \Sigma \lambda_i, \quad \lambda_B = \Sigma \lambda_j$

  Note that the reliability before fixes are made is

  $r(o) = \lambda_A + \lambda_B.$

- For each failure mode (λ_i), the effectiveness of the fix (d_i) is a known constant. (If a fix is known to reduce the failure rate by 1/2, d = 0.5).

- The average rate at which a new problem is discovered (H(t)) (not old problems reoccurring) follow a NHPP model of the same form as the AMSAA model, as follows:

  $h(t) = \alpha\beta\, T^{\beta-1}.$

  To estimate the reliability after the fixes are incorporated, first estimate β and α, as follows:

  $\hat{\beta} = \dfrac{M}{\sum_{i=1}^{M} \log(T/t_i)}$ where M is the number of failure modes,

  T is the cumulative time on test (this phase), and t_i is the cumulative test time at the ith failure.

  $\hat{\alpha} = \dfrac{M}{T^{\hat{\beta}}}, \; \bar{\beta} = \dfrac{M-1}{M}\hat{\beta}.$

  Using $\bar{\beta}$, compute $h(T) = \dfrac{M\bar{\beta}}{T}.$

  Compute the average of the effectiveness factors,

  $\hat{\mu}_d = \dfrac{1}{M} \cdot \sum_{i=1}^{M} d_i.$

  Compute the term $B(T) = \mu_d\, h(T)$.

  If N_A is the number of observed failures from type A modes, N_i is the number of observed failures from the ith type B mode, the new reliability can be computed as follows:

  $\bar{r}(T) = \dfrac{1}{T}\left(N_A + \sum_{i=1}^{M} (1-d_i)N_i\right) + B(t)$

  and the new MTBF = $1/\bar{r}(T)$.

To estimate the reliability jump as a result of delayed fixes from the EEC 103-2 CERT (Phase 2) to the next phase, break the failure data into failure modes, assign an effectiveness factor to fixes and estimate the current failure rate, as shown in Table D-11.

$$\hat{\beta} = \frac{10}{24.33} = 0.41$$

$$\bar{\beta} = \frac{9}{10} \cdot \hat{\beta} = 0.37$$

$$h(T) = \frac{(10 \cdot 0.37)}{50{,}000} = 0.074/1000$$

$$\hat{\mu}_d = 0.692$$

$$B(T) = \hat{\mu}_d\, h(T) = 0.05113/1000$$

$$\bar{r}(T) = \frac{1}{50{,}000}(3+5.25) + 0.05113/1000 = 0.000216$$

$$\text{MTBF} = 4627$$

Therefore, an MTBF of approximately 4600 hours is expected through the next phase.

TABLE D-11. ESTIMATE OF CURRENT FAILURE RATE

Modes	*Mode Type*	*Failure Rate*	N_i	d_i	*Cum Time of Failure*	T/t_i
1	B	1/10000	5	0.9	2.0	25000.0
2	A	1/50000	1	—	20.8	2404.0
3	A	1/50000	1	—	7274.0	6.87
4	B	14/50000	14	0.75	8996.3	5.56
5	B	1/25000	2	0.9	19988.7	2.501
6	B	1/25000	2	0.75	20751.8	2.410
7	B	1/50000	1	0.30	35342.5	1.415
8	A	1/50000	1	—	36082.7	1.386
9	B	1/50000	1	0.75	42586.2	1.174
10	B	1/50000	1	0.5	43059.4	1.161

E. INTERPRETATION OF RESULTS

1. Assessment

The point estimate of the instantaneous MTBF is the best estimate of the current MTBF. Upper and lower bounds on the MTBF show the possible variation which could have occurred with different hardware.

2. Prediction

Near term forecast can be made by extrapolating up the AMSAA model growth curve (the model fit within each test phase). Long term forecast can be made by projecting along the *ideal* or planned curve.

3. Control

In determining whether the program is on schedule, the planned reliability (considering the contribution of the present test phase) can be compared to the demonstrated reliability from assessment. If the interval estimate of MTBF covers the planned MTBF, the program is progressing within reasonable limits.

APPENDIX E

TIME SERIES ANALYSIS

A. INTRODUCTION

Many parametric and non-parametric methods for estimating the failure rate function have been developed. One disadvantage in these methods is the inflexibility built in by the assumed model and a lack of theory for forecasting.

The time series approach is free of any assumptions regarding either the failure distribution or the general form of its failure rate function. This approach is based on the concept that since the failure-rate function is a function of time, the estimated failure rates are constructed as being generated by a time series process. A Box-Jenkins time series model will be used to approximate the failure-rate process and to set confidence limits on these forecasts.

1. Motivation for Time Series

Suppose that $[t_i]_{i=1}^{n}$ is a specified sequence on the time axis, and let $\lambda_c(t)$ be the estimated cumulative failure rate function for the interval $[t_i, t_{i+1})$, where $t_i \leq t \leq t_{i+1}$. If $\lambda_c(t)$ depends only on observations in the interval $[t_i, t_{i+1})$, and those observations prior to t_i, then $[\ \lambda_c(t);\ t \geq 0\ ]$ is by definition a time series. If $\lambda_c(t)$ depended upon observations at time t_{i+1} or beyond t_{i+1}, then $[\ \lambda_c(t) : t \geq 0\ ]$ would not be a time series. If the time series $[\ \lambda_c(t) : t \geq 0\ ]$ can be made stationary (remain in equilibrium about a constant mean level) by various methods of Box-Jenkins, then the methods of Box-Jenkins can be used to forecast future failure rates.

2. Time Series Models and Notation

Let $\lambda_c(t), \lambda_c(t-1), \lambda_c(t-2), \ldots$ be the realizations of a stochastic process at equispaced time points $t, t-1, \ldots$ which are generated by a sequence of independent identically distributed random variables $a_t, a_{t-1}, \ldots$ with mean 0 and variance σ_a^2. This sequence of random variables is sometimes called a *white noise sequence*. The symbols a_t will be used for this type of random variable. The following sketch illustrates how the time series is generated.

White
Noise
a_t Linear Filter $\lambda_c(t)$

The white noise process a_t is transformed to the process $\lambda_c(t)$ by a linear filter. This linear filtering operation takes a weighted sum of previous white noise random variables to form $\lambda_c(t)$, as shown in the following equation:

$\lambda_c(t) = \mu + a_t + \psi_1 a_{t-1} + \psi_2 a_{t-2} \ldots$, where μ is the mean of the series. The sequence $\psi_1, \psi_2, \ldots$ form what is called the linear operator that transforms a_t into $\lambda_c(t)$.

a) General Form of the Time Series Model

A suitable model that describes many of the commonly occurring nonstationary time series process is the AutoRegressive Integrated Moving Average process of order p, d, q (ARIMA (p, d, q)). To illustrate the general form of the model, if $w_c^d(t)$ is the dth difference of $\lambda_c(t)$, (i.e., for $d = 1$, $w_c^1(t) = \lambda_c(t) - \lambda_c(t-1)$; for $d = 2$, $w_c^2(t) = w_c^1(t) - w_c^1(t-1) = \lambda_c(t) - \lambda_c(t-1) - (\lambda_c(t-1) - \lambda_c(t-2)) = \lambda_c(t) - 2\lambda_c(t-1) + \lambda_c(t-2)$; etc.), the form of the model is given in equation (1),

$$w_c^d(t) = \varphi_1 w_c^d(t-1) + \varphi_2 w_c^d(t-2) + \ldots + \varphi_p w_c^d(t-p) + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} \ldots - \theta_q a_{t-q} \quad (1)$$

where $\varphi_1, \varphi_2, \ldots \varphi_p$ are the p unknown autogressive parameters, and $\theta_1, \theta_2, \ldots \theta_q$ are the q unknown moving average parameters. Equation (1) can be simplified using the backward shift operator B defined by $B^k \cdot \lambda_c(t) = \lambda_c(t-k)$ where $k \geq 1$. For instance, $B^1 \cdot \lambda_c(t) = \lambda_c(t-1)$. We can rewrite the general ARIMA (p, d, q) model given equation (1) in terms of this backward notation given in equation (2):

$$\varphi(B)w_c^d(t) = \theta(B)a_t \tag{2}$$

where

$$\varphi(B) = 1 - \varphi_1 B^1 - \varphi_2 B^2 \ldots - \varphi_p B^p \quad \text{and}$$

$$\theta(B) = 1 - \theta_1 B^1 - \theta_2 B^2 \ldots - \theta_q B^q$$

A further generalization of the ARIMA (p, d, q) given in equation (2) is to include a deterministic polynomial trend, so that the model now becomes equation (3):

$$\varphi(B)w_c^d(t) = \theta_0 + \theta(B)a_t. \tag{3}$$

b) Specific Cases of the General Model

Many of the parameters of the general ARIMA (p, d, q) model will not be needed. If d = 0, then $w_c^0(t) = \lambda_c(t)$ and the process is known as an *autoregressive moving average* (ARIMA (p, 0, q)). The form of this model is given in equation (4).

$$\varphi(B)\,\lambda_c(t) = \theta(B)\,a_t \quad \text{or}$$

$$\lambda_c(t) - \varphi_1\,\lambda_c(t-1) - \varphi_2\,\lambda_c(t-2) \ldots - \varphi_p\,\lambda_c(t-p) = a_t - \theta_1\,a_{t-1} \ldots - \theta_q\,a_{t-q} \tag{4}$$

If p = 0, the time series is known as an *integrated moving average* (ARIMA (0, d, q)), whose form is given in equation (5):

$$w_c^d(t) = \theta(B)a_t \quad \text{or}$$

$$w_c^d(t) = a_t - \theta_1 a_{t-1} \ldots - \theta_q a_{t-q}. \tag{5}$$

If p = 0 and d = 0, the time series is known as a *moving average* (ARIMA (0, 0, q)), whose form is given in equation (6).

$$w_c^0(t) = \lambda_c(t) = \theta(B)a_t \quad \text{or}$$

$$\lambda_c(t) = a_t - \theta_1 a_{t-1} \ldots - \theta_q a_{t-q} \tag{6}$$

If q = 0 and d = 0, the time series is known as an *autoregressive* process (ARIMA (p, 0, 0)). The form of this model is given in equation (7).

$$\varphi(B)w_c^0(t) = \varphi(B)\lambda_c(t) = a_t \quad \text{or}$$

$$\lambda_c(t) - \varphi_1\lambda_c(t-1) - \varphi_2\lambda_c(t-2) \ldots - \varphi_p\lambda_c(t-p) = a_t \tag{7}$$

c) Model Constraints

The time series to be modeled must satisfy two requirements: invertibility and stationarity. To en ure invertibility and stationarity of the process, the parameters $\varphi_1, \varphi_2, \ldots \varphi_p$ and $\theta_1, \theta_2, \ldots \theta_q$ will need to fulfill certain requirements. The process of invertibility will now be explained by use of an example.

Consider the moving average process of order 1 (ARIMA (0, 0, 1): $\lambda_c(t) = a_t - \theta_1 a_{t-1} = (1 - \theta_1 B) a_t$. This moving average can be expressed as a weighted sum of previous failure rates, and a random element a_t, $\lambda_c(t) = a_t - \theta_1 \lambda_c(t-1) - \theta_1^2 \lambda_c(t-2) - \theta_1^3 \lambda_c(t-3) \ldots$ From the above equation, if $|\theta_1| \geq 1$,the weights associated with the $\lambda_c(t-i)$, $i = 1, 2, \ldots$, increases as i increases, which says that the process gives more emphasis to the observations which have occurred more and more into the past. The requirement that $|\theta_1| < 1$ is made, and the series is defined to be *invertible*. In general, it is a requirement that the roots of $\theta(B) = 0$ be less than one in absolute value for the process to be invertible. In the same manner, the time series will be stationary if the roots of $\varphi(B) = 0$ are less than one in absolute value.

d. The Recommended Time Series Model

From theory and practice, an integrated moving average process of order 1, (ARIMA (0, 1, 1)), is a satisfactory model for functions of the failure rate process. In practice, unless there is reason to believe the existence of a deterministic component, θ_0 in equation (3) is set equal to zero. Another satisfactory time series model is an ARIMA (0, 1, 0) model, $\lambda_c(t) - \lambda_c(t-1) = a_t$. Although the ARIMA (0, 1, 0) process fits the time series well, this model does not aid in the forecasting of future rates.

Characteristics of the Recommended Model

The form of the ARIMA (0, 1, 1) model used for the prediction of the $\lambda_c(t)$'s is written below in three equivalent forms.

$$\lambda_c(t) = \lambda_c(t-1) + a_t - \theta a_{t-1} \tag{8}$$

$$(1 - B)\,\lambda_c(t) = (1 - \theta B)a_t, \text{ since } B\cdot\lambda_c(t) = \lambda_c(t-1) \tag{9}$$

$$\varphi(B)\,\lambda_c(t) = \theta(B)a_t, \text{ where } \varphi(B) = 1 - B \text{ and } \theta(B) = (1 - \theta B). \tag{10}$$

The latter form, equation (10), will be used for purposes of simplicity. θ is a parameter to be estimated, and it may be recalled that $|\theta| < 1$ for purposes of invertibility.

For purposes of forecasting, it will be necessary to express the ARIMA (0, 1, 1) model as a linear function of the independent identically distributed random variables as follows:

$$\lambda_c(t) = a_t + \psi_1 a_{t-1} + \psi_2 a_{t-2} + \ldots \tag{11}$$

It can be easily verified that the $\psi_i = 1 - \theta$, $i = 1, 2, \ldots$ for the ARIMA (0, 1, 1) model so that equation (11) now becomes:

$$\psi_c(t) = a_t + (1 - \theta)a_{t-1} + (1 - \theta)a_{t-2} + \ldots \tag{12}$$

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The ARIMA (0, 1, 1) model may also be expressed as an infinite weighted sum of previous failure rates, $\lambda_c(t)$, plus a random component a_t:

$$\lambda_c(t) = \pi_1\lambda_c(t-1) + \pi_2\lambda_c(t-2) + \ldots + a_t \tag{13}$$

It can be verified that $\pi_1 = 1 - \theta$, $\pi_2 = \theta(1-\theta)$, $\pi_3 = \theta^2(1-\theta)$, . . . or in general $\pi_j = \theta^{j-1}(1-\theta)$, so that equation (13) now becomes:

$$\lambda_c(t) = (1-\theta)\ \lambda_c(t-1) + \theta(1-\theta)\ \lambda_c(t-2) + \theta^2\ (1-\theta)\ \lambda_c(t-3) + \ldots + a_t \tag{14}$$

Notice in the above equation, the failure rate at time t, $\lambda_c(t)$, depends less and less on past observations. Also, as θ approaches one, the weights die out more slowly.

3. Forecasting

Future failure rates will be predicted using the ARIMA (0, 1, 1) model. Suppose at time t, a future failure rate at time $t + \ell$, $\lambda_c(t+\ell)$, $\ell \geq 1$, shall be forecasted. At time $t + \ell$, the ARIMA (0, 1, 1) model may be written as:

$$\lambda_c(t+\ell) = \lambda_c(t+\ell-1) + a_{t+\ell} - \theta a_{t+\ell-1}. \tag{15}$$

Box-Jenkins shows that the minimum mean square error forecast at origin t, for lead time ℓ, is the conditional expectation of $\lambda_c(t+\ell)$ at time t. These conditional expectations at origin t are:

$$\lambda_c(t+1) = \lambda_c(t) - \theta a_t$$

$$\lambda_c(t+\ell) = \lambda_c(t+\ell-1), \ell \geq 2. \tag{16}$$

Hence, for all lead times, the forecasts at origin t, will follow a straight line parallel to the time axis.

The variance of the lead ℓ forecasts is needed so that confidence limits can be established. The variance of these forecasts at time origin t is given by equation (17):

$$\mathrm{Var}(\lambda_c(t+\ell)) = \sigma_a^2 \cdot (1 + (\ell-1)(1-\theta)^2)\text{, where } \sigma_a^2 \text{ is the variance of the } a_t\text{'s.} \tag{17}$$

Notice that the variance for the first forecast is σ_a^2, and this variance increases as the forecast leads are increased.

a) Model Identification

The first and most obvious way to start the process of model identification is to plot the cumulative failure rate, $\lambda_c(t)$, as a function of time. If the failure rates do not seem to be centered about a fixed mean, the data is probably nonstationary, and therefore the failure rates ($\lambda_c(t)$'s) should be differenced to produce the desired state of stationarity. Examination of the autocorrelations will also provide insight as to whether or not the time series is stationary or nonstationary. The autocorrelation function will now be defined.

Given a finite time series of cumulative failure rates, $\lambda_c(1), \lambda_c(2), \ldots \lambda_c(N)$ of N observations, the *autocovariance* at lag k, γ_k, is defined to be:

$$\gamma_k = \text{covariance } [\lambda_c(t), \lambda_c(t + k)]. \tag{18}$$

The autocorrelation at lag k, ρ_k, is defined to be

$$\rho_k = \frac{\gamma_k}{\gamma_0} \tag{19}$$

γ_k will be estimated by C_k, where

$$C_k = \frac{1}{N}\sum_{t=1}^{N-k} (\lambda_c(t) - \bar{\lambda}_c(t))(\lambda_c(t + k) - \bar{\lambda}_c(t)) \tag{20}$$

where $\bar{\lambda}_c(t)$ is the mean of the cumulative failure rates.

ρ_k will be estimated by r_k, where

$$r_k = \frac{C_k}{C_0}. \tag{21}$$

For a stationary time series, the estimated autocorrelation function, (r_k), for all lags should die out fairly rapidly. If the estimated autocorrelations do not die out rapidly, the time series should be differenced to obtain stationarity. The partial autocorrelation function will also provide insight as to whether the time series should be differenced. The idea of partial autocorrelation will be explained by an example. The simple correlation between two failure rates $\lambda_c(t)$ and $\lambda_c(t - 2)$ can perhaps be equal to zero, while the correlation between $\lambda_c(t)$ and $\lambda_c(t - 2)$ adjusted for $\lambda_c(t - 1)$ may not be zero. Hence, the partial autocorrelation between $\lambda_c(t)$ and $\lambda(t - 2)$, can be looked upon as the correlation between $\lambda_c(t)$ and $\lambda_c(t - 2)$ adjusted for $\lambda_c(t - 1)$. If the partial autocorrelation at lag 1 is close to unity, while the partial autocorrelations at lags greater than one are close to zero, this is evidence that the series should be differenced. Finally, the LJUNG modification of the Box-Pierce Q statistic produces a test whose distribution is Chi-square (χ^2) under the null hypothesis that the series is white noise. If this statistic is not statistically significant, and thus the process can be labeled as a white noise process, no differencing should be made, and the process can be modeled as $\lambda_c(t) = a_t$. However, if this test is not statistically significant, we can conclude that the series should be further modeled. From our suggested model, the cumulative failure rates should be differenced once, $\lambda_c(t) - \lambda_c(t - 1)$, although the above characteristics should be examined carefully to reach this conclusion.

Sometimes, the mean of the differenced series is not equal to zero, and an extra parameter θ_0, called a trend parameter should be included in the model. Usually we shall assume that this mean is equal to zero, or equivalently $\theta_0 = 0$, unless it is evident from the data that the problem has a nonzero mean. To statistically conclude if the mean of the differenced series is equal to zero, compute

$$\bar{w}_c^1(t) = \frac{\sum_{t=2}^{N} w_c^1(t)}{N - 1} \text{ where we recall that } w_c^1 = \lambda_c(t) - \lambda_c(t - 1) \tag{22}$$

and compare it with its approximate standard error given in equation (23).

$$\sigma(\bar{w}_c^1(t)) = \left(\frac{C_0(1 + 2r_1)}{N - 1}\right)^{1/2}. \tag{23}$$

After deciding that the series should be differenced once and a test for a trend parameter has been made, examination of the autocorrelations and partial autocorrelations for the differenced series, $\lambda_c(t) - \lambda_c(t-1)$, should be made. For the suggested ARIMA (0, 1, 1) model, the autocorrelations should drop off after the first lag, and the partial autocorrelations should tail off exponentially. To ensure that the residuals are white noise, the LJUNG modification of the Box-Pierce Q statistic should be performed again, after the ARIMA (0, 1, 1) model has been identified. This statistic is calculated on the model residuals to test the hypothesis that they are white noise.

b) Parameter Estimation

Assuming that the time series model is an ARIMA (0, 1, 1), use of the relation of the moving average parameter to the first lag autocorrelation ρ_1, is given in the equation below:

$$\rho_1 = \frac{-\theta}{1-\theta^2} \tag{24}$$

To obtain an estimate for θ, solve the above equation for θ. Two possible solutions are

$$\theta = -\frac{1}{2\rho_1} + \left(\frac{1}{(2\rho_1)^2} - 1\right)^{1/2} \tag{25}$$

and

$$\theta = -\frac{1}{2\rho_1} - \left(\frac{1}{(2\rho_1)^2} - 1\right)^{1/2}. \tag{26}$$

Only one of the above possible solutions can satisfy the invertibility condition that $-1 < \theta < 1$. Estimating ρ_1 by r_1, and substituting into the above equation will yield a preliminary estimate for θ. Nonlinear estimation will improve upon this estimate, until a suitable solution has been found.

The final parameter to be estimated is the variance of the a_t's, which is σ_a^2. An estimate of σ_a^2 may be obtained by substituting the preliminary estimate of θ and γ_0 by its estimate C_0 in to equation (27) and solving for σ_a^2

$$\gamma_0 = \sigma_a^2(1 + \theta^2) \tag{27}$$

B. PROCEDURE FOR IDENTIFYING TIME SERIES MODELS

The F100 electronic engine control (EEC) data generated over 4 years of development will be used as an illustrative example. The model form and parameters will be derived using the time series methods in Box-Jenkins. The computer software used to evaluate the data is the Statistical Analysis System (SAS Institute Inc., Cary, North Carolina).

1. Model Identification

a) Differencing the Time Series

The first step in trying to identify the form of the model is to plot the original data of the cumulative failure rates against time beginning at the first failure. The EEC data does not seem to be fluctuating about a fixed level, as shown in Figure E-1. This is one indication that the series should be differenced to remove the changing shifts in level. This is one type of nonstationary

time series. Another indication that the series should be differenced is to calculate the estimated autocorrelations and estimated partial autocorrelations of the undifferenced series. For the EEC data, the estimated autocorrelations follow a gentle linear decline, thus suggesting that the series is nonstationary, as opposed to a rapid geometric decline of the autocorrelations for a stationary series (see Computer Output 1). The estimated partial autocorrelation shows a correlation approaching one at the first lag, and a correlation near zero at all lags greater than one (see Computer Output 2). This indicates an autoregressive model with the parameter θ approaching one, again indicating that the series should be differenced. For the undifferenced EEC data, the autocorrelation check for white noise is significant so that the hypothesis that the series is white noise is rejected. (see Computer Output 3). Hence, there is strong evidence that the process is nonstationary.

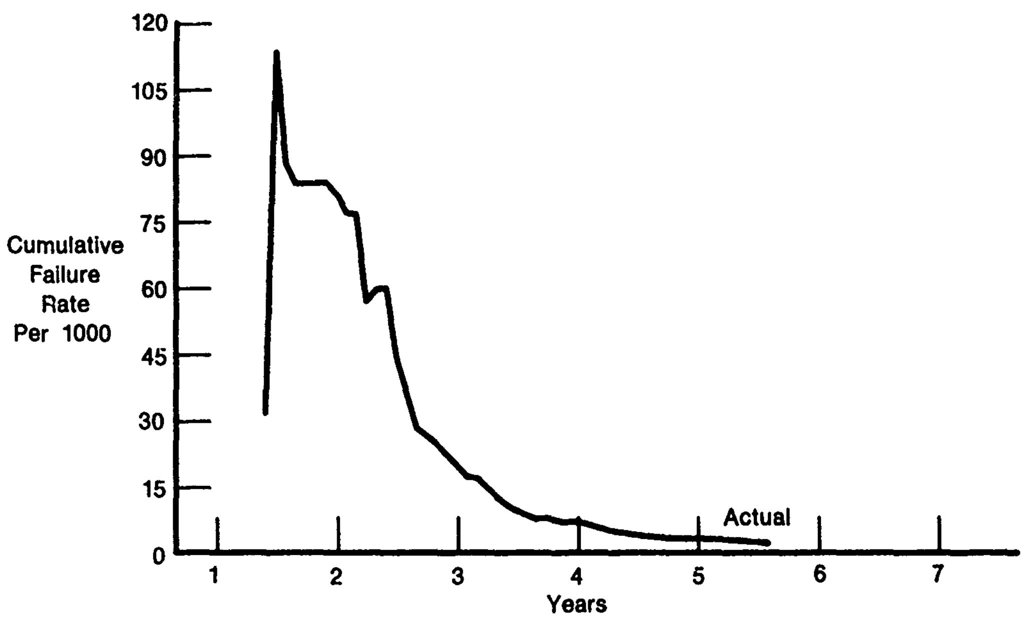

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Figure E-1. Cumulative Failure Rate of Undifferenced EEC Data

After differencing the series once ($w_c^1(t) = \lambda_c(t) - \lambda_c(t-1)$), a plot of this first difference should be made to indicate whether the process has obtained stationarity. For the EEC data, the first difference seems to have made the series stable (as graphically shown in Figure E-2). Calculation of the estimated autocorrelations and estimated partial autocorrelations should further aid in this decision. The autocorrelations for the one-differenced EEC data are close to zero for all lags greater than zero, as opposed to the linear decline of the autocorrelations for the undifferenced series (see Computer Output 4). Notice also that the partial autocorrelations for the first lag is not close to one, as opposed to the partial autocorrelations approaching one for the first lag of the undifferenced series (see Computer Output 5). Hence, there is strong evidence that the series has obtained stationarity by differencing the series once.

The variance estimate of the series σ_a^2 has decreased dramatically by the differencing procedure (0.000964022 to 0.000162855) as expected (see Computer Output 6 and Computer Output 7, respectfully). Overdifferencing the series will increase this variance. As an example, suppose the EEC data had been differenced twice ($w_c^2(t)$). Notice that the variance of the series increased from 0.000162855 to 0.000265891, indicating another reason that the series should not be second differenced (see Computer Output 7 and Computer Output 8, respectfully).

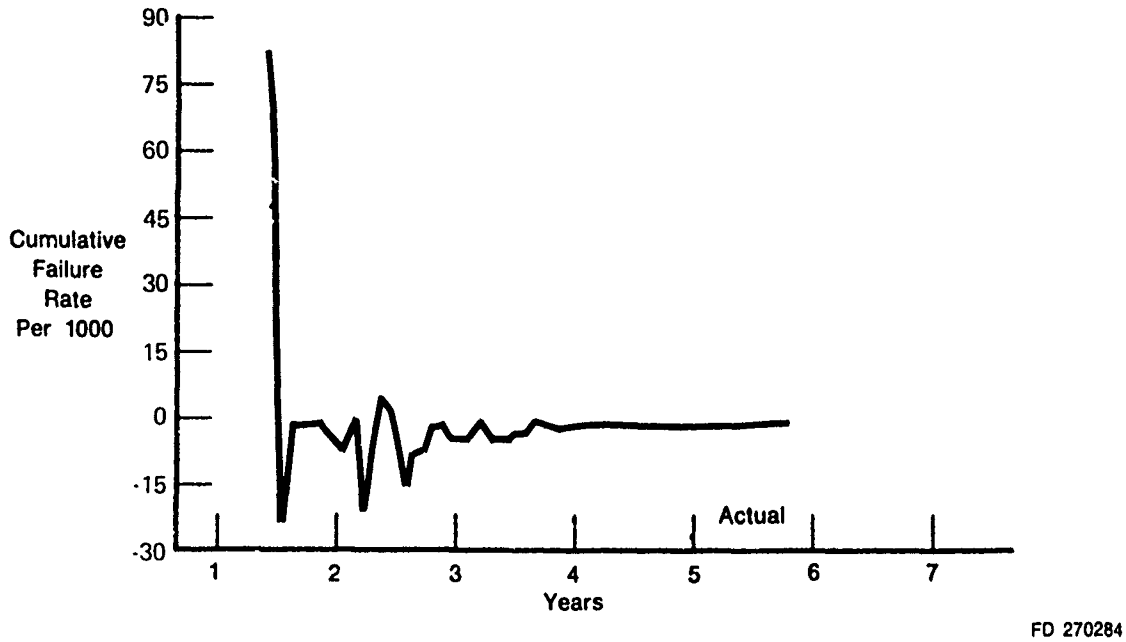

Figure E-2. Cumulative Failure Rate of First Differenced EEC Data

b) Examination of the Autocorrelation Functions

Now that the series has obtained stationarity through first differencing, further examination of the autocorrelation and partial autocorrelation functions should be done. The Statistical Analysis System (SAS) computes these functions, as well as an inverse autocorrelation function, whose behavior is somewhat similar to the partial autocorrelation function. Table E-1 is a chart of the behavior of the estimated functions after stationarity has been obtained.

TABLE E-1. ESTIMATED FUNCTIONS BEHAVIOR AFTER STATIONARITY HAS BEEN OBTAINED

	Moving Average of Order Q	*Autoregressive of Order P*	*Autoregressive Moving Average of Order P, Q*	*White Noise*
Autocorrelation Function	D(Q)	T	T	0
Partial Autocorrelation Function	T	D(P)	T	0
Inverse Autocorrelation Function	T	D(P)	T	0

Key: D(Q) means the function drops off to zero after lag Q.
D(P) means the function drops off to zero after lag P.
T means the function tails of exponentially.
0 means the function is zero at all nonzero lags.

Examining the estimated autocorrelations for the once-differenced EEC data reveals a slight drop off to zero after lag one, although the autocorrelation does in fact lie within two standard errors of zero (see Computer Output 4). The estimated inverse autocorrelations and estimated partial autocorrelations show a slight decline towards zero, beginning with the first lag (see Computer Output 5). These statistics indicate that the differenced series could be a moving

average of order 1 ($\lambda_c(t) - \lambda_c(t-1) = a_t - \theta a_{t-1}$) or a white noise sequence ($\lambda_c(t) - \lambda_c(t-1) = a_t$). The autocorrelation check for white noise indicates that the hypothesis that the differenced series is a white noise process cannot be rejected (see Computer Output 9). However, to obtain information to aid the process of forecasting, a moving average parameter will be added to the statistical model. After adding this extra term to the once-differenced EEC data, a significant moving average parameter was obtained; and, in adding this extra term, the residuals are statistically white noise by observing the autocorrelation check of residuals (see Computer Output 10). The estimate for this moving average parameter is 0.225118 (see Computer Output 10). A trend parameter could be necessary in the model. Testing for the significance of this parameter yields a nonsignificant t statistic (−0.33). Thus, a trend parameter is unnecessary in the time series model (see Computer Output 11). The final time series model is $\lambda_c(t) - \lambda_c(t-1) = a_t - 0.225118\, a_{t-1}$ with $\sigma_a^2 = 0.000158384$.

2. Forecasting

For the forecasted value for January of the sixth year, using the forecasting equation $\lambda_c(t+1) = \lambda_c(t) - \theta a_t$, at t = 51, the forecasted value is $\lambda_c(51) = 0.0020$ (see Computer Output 12 and 13). Consequently, since $\lambda_c(t + \ell) = \lambda_c(t + \ell - 1)$ for $\ell \geq 2$, $\lambda_c(51) = \lambda_c(52) = \ldots = 0.0020$. The predicted cumulative failure rate is a straight line parallel to the time axis. The variance of these forecasts increase with time according to the equation, Var $(\lambda_c(t + \ell)) = \sigma_a^2 (1 + (\ell - 1)(1 - \theta)^2)$. Thus, the variance for $\lambda_c(51) = \sigma_a^2 = 0.000158384$, and the standard deviation is 0.0126. This variance increases as the forecast in time increase (see Computer Output 12 and 13). The 1 year predicted cumulative failure rate is a line parallel to the time axis, as shown in Figure E-3), and no cumulative failure rate decline is predicted.

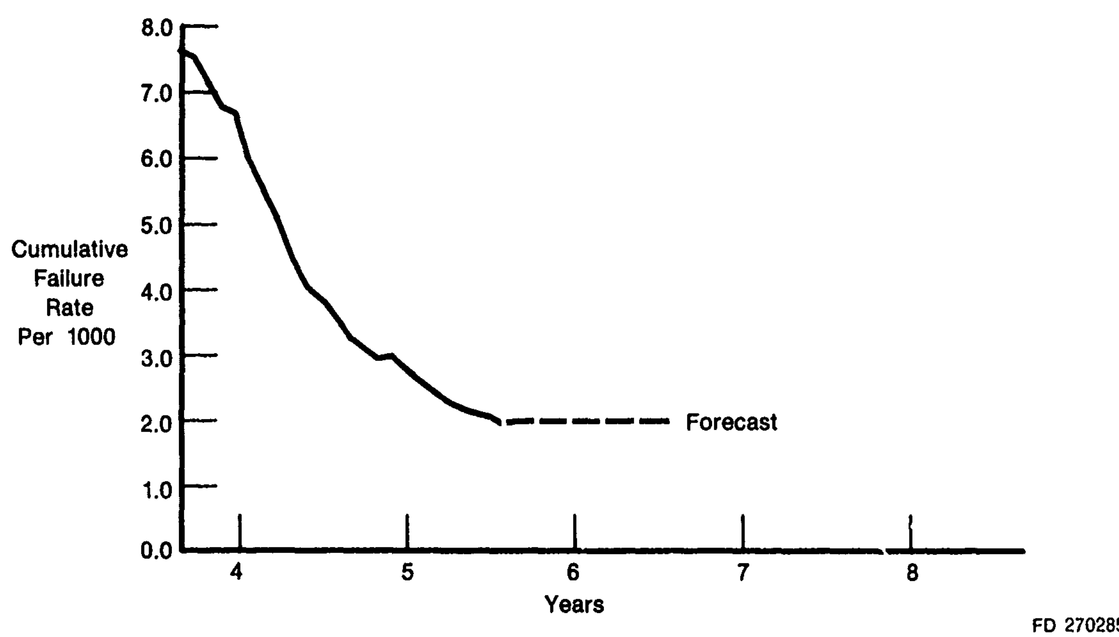

Figure E-3. Cumulative Failure Rate of EEC Data With Prediction

COMPUTER OUTPUT 1 THROUGH 13

COMPUTER OUTPUT 1

```
TIME SERIES ANALYSIS OF EEC DATA                16:29 TUESDAY, OCTOBER 4, 1983    2
             ARIMA PROCEDURE

NAME OF VARIABLE        =  RATE

MEAN OF WORKING SERIES= 0.0267515
STANDARD DEVIATION     = 0.0310487
NUMBER OF OBSERVATIONS=        51

                    AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1       STD
  0 .000964022     1.00000 |                    |********************|         0
  1 .000877744     0.91050 |              .     |******************  |  0.140028
  2 .000817584     0.84310 |           .        |*****************   |  0.228294
  3 0.00076357     0.79207 |         .          |****************    |  0.283416
  4 .000701892     0.72809 |       .            |***************     |  0.323925
  5 0.00063532     0.65903 |      .             |*************.      |  0.354565
  6 .000561109     0.58205 |     .              |************  .     |  0.377821
  7 .000485586     0.50371 |    .               |**********     .    |  0.395011
  8 .000411434     0.42679 |    .               |*********      .    |  0.407411
  9 .000326337     0.33852 |   .                |*******         .   |  0.416085
 10 .000272756     0.28294 |   .                |******          .   |  0.421451
 11 .000212347     0.22027 |   .                |****            .   |  0.425159
 12 .000141291     0.14656 |   .                |***             .   |  0.427391
 13 .000088565     0.09187 |   .                |**              .   |  0.428375
 14 .000046002     0.04772 |   .                |*               .   |  0.428761
 15 .000013857     0.01437 |   .                |                .   |  0.428865
 16 -1.688E-05    -0.01751 |   .                |                .   |  0.428875
 17 -4.670E-05    -0.04844 |   .               *|                .   |  0.428839
 18 -7.433E-05    -0.07710 |   .              **|                .   |  0.428996
 19 -0.0000995    -0.10321 |   .              **|                .   |  0.429268
 20 -.00012158    -0.12611 |   .             ***|                .   |  0.429754
 21 -.00014358    -0.14894 |   .             ***|                .   |  0.430479
 22 -.00016454    -0.17068 |   .             ***|                .   |  0.431489
 23 -.00018386    -0.19072 |   .            ****|                .   |   0.43281
 24 -.00020134    -0.20885 |   .            ****|                .   |  0.434455
                           '.' MARKS TWO STANDARD ERRORS
```

COMPUTER OUTPUT 2


```
                  INVERSE AUTOCORRELATIONS

LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  1    -0.42544  |           *********|     .              |
  2    -0.03562  |              .    *|     .              |
  3    -0.01426  |              .     |     .              |
  4    -0.01909  |              .     |     .              |
  5    -0.01667  |              .     |     .              |
  6    -0.00203  |              .     |     .              |
  7     0.02477  |              .     |     .              |
  8    -0.12127  |              .   **|     .              |
  9     0.20077  |              .     |**** .              |
 10    -0.07923  |              .   **|     .              |
 11    -0.09523  |              .   **|     .              |
 12     0.09512  |              .     |**   .              |
 13     0.00018  |              .     |     .              |
 14     0.02297  |              .     |     .              |
 15    -0.01677  |              .     |     .              |
 16     0.00046  |              .     |     .              |
 17    -0.03394  |              .    *|     .              |
 18     0.03164  |              .     |*    .              |
 19    -0.00019  |              .     |     .              |
 20    -0.04359  |              .    *|     .              |
 21     0.03061  |              .     |*    .              |
 22     0.00002  |              .     |     .              |
 23    -0.00735  |              .     |     .              |
 24     0.01017  |              .     |     .              |
```

```
                  PARTIAL AUTOCORRELATIONS

LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  1     0.91050  |              .     |******************  |
  2     0.11161  |              .     |**   .              |
  3     0.02628  |              .     |*    .              |
  4    -0.06420  |              .    *|     .              |
  5    -0.07660  |              .   **|     .              |
  6    -0.10315  |              .   **|     .              |
  7    -0.07549  |              .   **|     .              |
  8    -0.05189  |              .    *|     .              |
  9    -0.11867  |              .   **|     .              |
 10     0.11964  |              .     |**   .              |
 11    -0.03344  |              .    *|     .              |
 12    -0.09951  |              .   **|     .              |
 13     0.02891  |              .     |*    .              |
 14     0.03097  |              .     |*    .              |
 15     0.03993  |              .     |*    .              |
 16    -0.01006  |              .     |     .              |
 17    -0.02506  |              .    *|     .              |
 18    -0.06246  |              .    *|     .              |
 19    -0.01501  |              .     |     .              |
 20    -0.03011  |              .    *|     .              |
 21    -0.07249  |              .    *|     .              |
 22    -0.01632  |              .     |     .              |
 23    -0.01591  |              .     |     .              |
 24    -0.01763  |              .     |     .              |
```

COMPUTER OUTPUT 3

TIME SERIES ANALYSIS OF EEC DATA 14:31 THURSDAY, OCTOBER 6, 1983 4

AUTOCORRELATION CHECK FOR WHITE NOISE

TO LAG	CHI SQUARE	DF	PROB	AUTOCORRELATIONS					
6	196.18	6	0.000	0.911	0.848	0.792	0.728	0.659	0.582
12	240.64	12	0.000	0.504	0.427	0.339	0.283	0.220	0.147
18	242.12	18	0.000	0.092	0.048	0.014	-0.018	-0.048	-0.077
24	257.00	24	0.000	-0.103	-0.126	-0.149	-0.171	-0.191	-0.209

COMPUTER OUTPUT 4

```
                     TIME SERIES ANALYSIS OF EEC DATA               16:29 TUESDAY, OCTOBER 4, 1983    5
                             ARIMA PROCEDURE

                    NAME OF VARIABLE       =  RATE
                    PERIODS OF DIFFERENCING=  1.

                    MEAN OF WORKING SERIES=-.00059741
                    STANDARD DEVIATION     = 0.0127615
                    NUMBER OF OBSERVATIONS=         50

                              AUTOCORRELATIONS

LAG COVARIANCE CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1       STD
  0 .000162855    1.00000   |                   |********************|          0
  1 -3.625E-05   -0.22256   |             . ****|     .              |   0.141421
  2 -4.692E-06   -0.02881   |             .    *|     .              |   0.148261
  3  7.350E-06    0.04513   |             .     |*    .              |   0.148373
  4  4.048E-06    0.02486   |             .     |     .              |   0.148647
  5  6.854E-06    0.04209   |             .     |*    .              |    0.14873
  6  4.059E-07    0.00249   |             .     |     .              |   0.148968
  7 -2.017E-06   -0.01238   |             .     |     .              |   0.148969
  8 .000010717    0.06581   |             .     |*    .              |    0.14899
  9 -3.305E-05   -0.20294   |             . ****|     .              |    0.14957
 10 0.00000839    0.05152   |             .     |*    .              |   0.154979
 11  9.875E-06    0.06064   |             .     |*    .              |   0.155322
 12 -1.934E-05   -0.11874   |             .   **|     .              |   0.155794
 13 -9.199E-06   -0.05649   |             .    *|     .              |   0.157594
 14 -1.012E-05   -0.06215   |             .    *|     .              |   0.157998
 15 -8.725E-07   -0.00536   |             .     |     .              |   0.158437
 16 -8.380E-07   -0.00515   |             .     |     .              |    0.15849
 17 -2.155E-06   -0.01323   |             .     |     .              |   0.158494
 18 -2.332E-06   -0.01432   |             .     |     .              |   0.158516
 19 -2.864E-06   -0.01759   |             .     |     .              |   0.158542
 20 -4.852E-08   -0.00030   |             .     |     .              |   0.158581
 21 -1.053E-06   -0.00646   |             .     |     .              |   0.158581
 22 -1.609E-06   -0.00988   |             .     |     .              |   0.158586
 23 -1.849E-06   -0.01135   |             .     |     .              |   0.158598
 24 -2.194E-06   -0.01347   |             .     |     .              |   0.158614
                            '.' MARKS TWO STANDARD ERRORS
```

Computer Output 5

```
                 TIME SERIES ANALYSIS OF EEC DATA              16:29 TUESDAY, OCTOBER 4, 1983

                           INVERSE AUTOCORRELATIONS

LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  1     0.28753  |             .     |******             |
  2     0.10007  |             .     |**   .             |
  3     0.01567  |             .     |     .             |
  4    -0.03339  |             .    *|     .             |
  5    -0.03958  |             .    *|     .             |
  6    -0.01579  |             .     |     .             |
  7     0.01282  |             .     |     .             |
  8     0.02999  |             .     |*    .             |
  9     0.19325  |             .     |**** .             |
 10     0.05364  |             .     |*    .             |
 11     0.03504  |             .     |*    .             |
 12     0.16266  |             .     |***  .             |
 13     0.15112  |             .     |***  .             |
 14     0.12547  |             .     |***  .             |
 15     0.05422  |             .     |*    .             |
 16     0.00743  |             .     |     .             |
 17    -0.01222  |             .     |     .             |
 18     0.01946  |             .     |     .             |
 19     0.00679  |             .     |     .             |
 20     0.00687  |             .     |     .             |
 21     0.06678  |             .     |*    .             |
 22     0.06983  |             .     |*    .             |
 23     0.05717  |             .     |*    .             |
 24     0.03514  |             .     |*    .             |


                           PARTIAL AUTOCORRELATIONS

LAG CORRELATION -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1
  1    -0.22256  |             . ****|     .             |
  2    -0.08243  |             .   **|     .             |
  3     0.02103  |             .     |     .             |
  4     0.04039  |             .     |*    .             |
  5     0.06504  |             .     |*    .             |
  6     0.03101  |             .     |*    .             |
  7    -0.00319  |             .     |     .             |
  8     0.06078  |             .     |*    .             |
  9    -0.19246  |             . ****|     .             |
 10    -0.04117  |             .    *|     .             |
 11     0.04284  |             .     |*    .             |
 12    -0.08870  |             .   **|     .             |
 13    -0.09533  |             .   **|     .             |
 14    -0.10210  |             .   **|     .             |
 15    -0.04614  |             .    *|     .             |
 16    -0.02203  |             .     |     .             |
 17     0.01959  |             .     |     .             |
 18    -0.03299  |             .    *|     .             |
 19    -0.01321  |             .     |     .             |
 20     0.02367  |             .     |     .             |
 21    -0.04145  |             .    *|     .             |
 22    -0.04875  |             .    *|     .             |
 23    -0.04625  |             .    *|     .             |
 24    -0.04196  |             .    *|     .             |
```

COMPUTER OUTPUT 6

ARIMA: PRELIMINARY ESTIMATION

WHITE NOISE VARIANCE EST=.000964022

ARIMA: LEAST SQUARES ESTIMATION

ITERATION	SSE	LAMBDA
0	0.085663	1.0E-05
1	0.085663	1.0E-06

PARAMETER ESTIMATE STD ERROR T RATIO LAG

VARIANCE ESTIMATE =0.00167967
STD ERROR ESTIMATE = 0.0409837
NUMBER OF RESIDUALS= 51

CORRELATIONS OF THE ESTIMATES

AUTOCORRELATION CHECK OF RESIDUALS

TO LAG	CHI SQUARE	DF	PROB	AUTOCORRELATIONS					
6	220.99	6	0.000	0.946	0.883	0.831	0.776	0.718	0.655
12	301.29	12	0.000	0.592	0.530	0.463	0.414	0.368	0.315
18	321.93	18	0.000	0.272	0.240	0.217	0.197	0.178	0.160
24	329.36	24	0.000	0.145	0.132	0.120	0.108	0.098	0.090

Computer Output 7


```
        ARIMA: PRELIMINARY ESTIMATION

 WHITE NOISE VARIANCE EST=.000162855

        ARIMA: LEAST SQUARES ESTIMATION

            ITERATION    SSE      LAMBDA
              0      0.0081606   1.0E-05
              1      0.0081606   1.0E-06

PARAMETER     ESTIMATE     STD ERROR    T RATIO   LAG


VARIANCE  ESTIMATE =.000163212
STD ERROR ESTIMATE = 0.0127754
NUMBER OF RESIDUALS=    50

          CORRELATIONS OF THE ESTIMATES



        AUTOCORRELATION CHECK OF RESIDUALS

 TO    CHI                               AUTOCORRELATIONS
LAG  SQUARE DF   PROB
  6    2.79  6  0.835      -0.214 -0.022  0.051  0.031  0.048  0.009
 12    6.94 12  0.862      -0.006  0.071 -0.197  0.056  0.065 -0.114
 18    7.41 18  0.996      -0.053 -0.059 -0.003 -0.003 -0.011 -0.012
 24    7.46 24  0.999      -0.016  0.001 -0.005 -0.008 -0.010 -0.012
```

Computer Output 8

```
TIME SERIES ANALYSIS OF EEC DATA              14:29 THURSDAY, OCTOBER 6, 1983   13

         ARIMA: PRELIMINARY ESTIMATION

   WHITE NOISE VARIANCE EST=.000265891

         ARIMA: LEAST SQUARES ESTIMATION

             ITERATION    SSE   LAMBDA
                0   0.0131646  1.0E-05
                1   0.0131646  1.0E-06

PARAMETER    ESTIMATE    STD ERROR    T RATIO   LAG


VARIANCE  ESTIMATE =.000268665
STD ERROR ESTIMATE =  0.016391
NUMBER OF RESIDUALS=   49

          CORRELATIONS OF THE ESTIMATES



         AUTOCORRELATION CHECK OF RESIDUALS

 TO   CHI                                  AUTOCORRELATIONS
LAG  SQUARE DF   PROB
  6    2.58  6  0.859       -0.209 -0.055  0.029 -0.023  0.035  0.001
 12    8.94 12  0.708       -0.050  0.194 -0.201  0.006  0.135 -0.052
 18    9.09 18  0.958       -0.002 -0.046 -0.000  0.005  0.001 -0.000
 24    9.11 24  0.997       -0.013  0.005  0.002  0.002  0.000 -0.009
```

Computer Output 9

TIME SERIES ANALYSIS OF EEC DATA 14:31 THURSDAY, OCTOBER 6, 1983 8

AUTOCORRELATION CHECK FOR WHITE NOISE

TO LAG	CHI SQUARE	DF	PROB	AUTOCORRELATIONS					
6	2.92	6	0.818	-0.223	-0.029	0.045	0.025	0.042	0.002
12	7.20	12	0.844	-0.012	0.066	-0.203	0.052	0.061	-0.119
18	7.73	18	0.982	-0.056	-0.062	-0.005	-0.005	-0.013	-0.014
24	7.80	24	0.999	-0.018	-0.000	-0.006	-0.010	-0.011	-0.013

Computer Output 10

ARIMA: PRELIMINARY ESTIMATION

MOVING AVERAGE ESTIMATES
1 0.22256

WHITE NOISE VARIANCE EST=.000155169

ARIMA: LEAST SQUARES ESTIMATION

ITERATION	SSE	MA1,1	LAMBDA
0	0.00776009	0.222562	1.0E-05
1	0.00776084	0.225274	1.0E-06
2	0.00776084	0.225118	1.0E-07

PARAMETER	ESTIMATE	STD ERROR	T RATIO	LAG
MA1,1	0.225118	0.139147	1.62	1

VARIANCE ESTIMATE =.000158384
STD ERROR ESTIMATE = 0.0125851
NUMBER OF RESIDUALS= 50

CORRELATIONS OF THE ESTIMATES

	MA1,1
MA1,1	1.000

AUTOCORRELATION CHECK OF RESIDUALS

TO LAG	CHI SQUARE	DF	PROB	AUTOCORRELATIONS					
6	0.69	5	0.983	-0.002	-0.010	0.062	0.058	0.066	0.024
12	4.16	11	0.965	0.006	0.033	-0.184	0.026	0.044	-0.126
18	5.48	17	0.996	-0.098	-0.085	-0.024	-0.012	-0.018	-0.021
24	5.60	23	1.000	-0.021	-0.005	-0.009	-0.014	-0.017	-0.017

COMPUTER OUTPUT 11

```
        TIME SERIES ANALYSIS OF EEC DATA              16:29 TUESDAY, OCTOBER 4, 1983   18

         ARIMA: PRELIMINARY ESTIMATION

   CONSTANT TERM ESTIMATE=-.00059741
   WHITE NOISE VARIANCE EST=.000162855

        ARIMA: LEAST SQUARES ESTIMATION

   ITERATION   SSE      MU        CONSTANT    LAMBDA
      0  0.00814275  -6.0E-04 -.00059741  1.0E-05
      1  0.00814275  -6.0E-04 -.00059741  1.0E-06

 PARAMETER    ESTIMATE     STD ERROR   T RATIO  LAG

 MU          -.00059741   0.00182306     -0.33   0

 CONSTANT ESTIMATE  =-.00059741

 VARIANCE  ESTIMATE =.000166179
 STD ERROR ESTIMATE =  0.012891
 NUMBER OF RESIDUALS=   50

          CORRELATIONS OF THE ESTIMATES

                   MU
          MU       1.000


        AUTOCORRELATION CHECK OF RESIDUALS

 TO   CHI                          AUTOCORRELATIONS
LAG  SQUARE DF   PROB
  6    2.92  5  0.712      -0.223 -0.029  0.045  0.025  0.042  0.002
 12    7.20 11  0.783      -0.012  0.066 -0.203  0.052  0.061 -0.119
 18    7.73 17  0.972      -0.056 -0.062 -0.005 -0.005 -0.013 -0.014
 24    7.80 23  0.999      -0.018 -0.000 -0.006 -0.010 -0.011 -0.0[illegible]
```

COMPUTER OUTPUT 12

FORECASTS FOR VARIABLE RATE

OBS	FORECAST	STD ERROR	LOWER 95%	UPPER 95%	ACTUAL	RESIDUAL
2	0.0318	0.0126	0.0072	0.0565	0.1134	0.0815
3	0.0950	0.0126	0.0704	0.1197	0.0880	-0.0070
4	0.0896	0.0126	0.0649	0.1143	0.0833	-0.0063
5	0.0847	0.0126	0.0601	0.1094	0.0833	-0.0014
6	0.0837	0.0126	0.0590	0.1083	0.0833	-0.0003
7	0.0834	0.0126	0.0587	0.1081	0.0833	-0.0001
8	0.0833	0.0126	0.0587	0.1080	0.0806	-0.0027
9	0.0813	0.0126	0.0566	0.1059	0.0768	-0.0044
10	0.0778	0.0126	0.0532	0.1025	0.0763	-0.0015
11	0.0767	0.0126	0.0520	0.1013	0.0562	-0.0205
12	0.0608	0.0126	0.0361	0.0855	0.0593	-0.0015
13	0.0596	0.0126	0.0350	0.0843	0.0594	-0.0002
14	0.0595	0.0126	0.0348	0.0841	0.0439	-0.0156
15	0.0474	0.0126	0.0227	0.0720	0.0353	-0.0121
16	0.0380	0.0126	0.0133	0.0627	0.0279	-0.0101
17	0.0302	0.0126	0.0055	0.0549	0.0262	-0.0040
18	0.0271	0.0126	0.0024	0.0518	0.0244	-0.0027
19	0.0250	0.0126	0.0004	0.0497	0.0219	-0.0032
20	0.0226	0.0126	-0.0021	0.0472	0.0195	-0.0030
21	0.0202	0.0126	-0.0045	0.0449	0.0172	-0.0030
22	0.0179	0.0126	-0.0068	0.0425	0.0164	-0.0015
23	0.0167	0.0126	-0.0079	0.0414	0.0149	-0.0018
24	0.0153	0.0126	-0.0093	0.0400	0.0130	-0.0023
25	0.0136	0.0126	-0.0111	0.0382	0.0112	-0.0024
26	0.0117	0.0126	-0.0130	0.0364	0.0094	-0.0023
27	0.0099	0.0126	-0.0148	0.0346	0.0086	-0.0013
28	0.0089	0.0126	-0.0158	0.0336	0.0076	-0.0013
29	0.0079	0.0126	-0.0167	0.0326	0.0075	-0.0004
30	0.0076	0.0126	-0.0170	0.0323	0.0072	-0.0004
31	0.0073	0.0126	-0.0174	0.0320	0.0068	-0.0005
32	0.0069	0.0126	-0.0178	0.0316	0.0067	-0.0002
33	0.0067	0.0126	-0.0179	0.0314	0.0060	-0.0007
34	0.0062	0.0126	-0.0185	0.0308	0.0055	-0.0006
35	0.0057	0.0126	-0.0190	0.0303	0.0050	-0.0006
36	0.0052	0.0126	-0.0195	0.0298	0.0045	-0.0007
37	0.0046	0.0126	-0.0200	0.0293	0.0041	-0.0006
38	0.0042	0.0126	-0.0205	0.0288	0.0038	-0.0003
39	0.0039	0.0126	-0.0207	0.0286	0.0035	-0.0004
40	0.0036	0.0126	-0.0210	0.0283	0.0033	-0.0003
41	0.0034	0.0126	-0.0213	0.0280	0.0031	-0.0002
42	0.0032	0.0126	-0.0215	0.0278	0.0030	-0.0002
43	0.0030	0.0126	-0.0217	0.0277	0.0030	-0.0000
44	0.0030	0.0126	-0.0217	0.0277	0.0028	-0.0002
45	0.0028	0.0126	-0.0218	0.0275	0.0026	-0.0002
46	0.0027	0.0126	-0.0220	0.0273	0.0025	-0.0002
47	0.0025	0.0126	-0.0222	0.0272	0.0023	-0.0002
48	0.0024	0.0126	-0.0223	0.0270	0.0022	-0.0001
49	0.0023	0.0126	-0.0224	0.0269	0.0021	-0.0001
50	0.0022	0.0126	-0.0225	0.0268	0.0021	-0.0001
51	0.0021	0.0126	-0.0226	0.0267	0.0020	-0.0001
--------FORECAST BEGINS---------						
52	0.0020	0.0126	-0.0227	0.0267		
53	0.0020	0.0159	-0.0292	0.0332		
54	0.0020	0.0187	-0.0346	0.0386		

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TIME SERIES ANALYSIS OF EEC DATA 16:29 TUESDAY, OCTOBER 4, 1983 28

FORECASTS FOR VARIABLE RATE

OBS	FORECAST	STD ERROR	LOWER 95%	UPPER 95%	ACTUAL	RESIDUAL
55	0.0020	0.0211	-0.0393	0.0433		
56	0.0020	0.0232	-0.0435	0.0475		
57	0.0020	0.0252	-0.0473	0.0513		
58	0.0020	0.0270	-0.0509	0.0549		
59	0.0020	0.0287	-0.0543	0.0583		
60	0.0020	0.0303	-0.0574	0.0614		
61	0.0020	0.0318	-0.0604	0.0644		
62	0.0020	0.0333	-0.0633	0.0673		
63	0.0020	0.0347	-0.0660	0.0700		

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